The Divisia approach to measuring output and productivity:
with an application to the BEA/BLS integrated industry-level production account, 1987-2020

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ROADMAP

- Two approaches to index numbers of quantities and prices:

  (1) the Divisia approach; (2) the approach of most NSIs
- Review of Divisia index numbers and of superlative index numbers
- Questions:
  1. How close do real life indices get to the desirable properties of Divisia indices?
  2. How different are chain-linked indices from alternative non-chained indices (such as 2-year indices)?
- Use the BEA/BLS integrated industry-level production account to answer these questions.

Jorgenson and Griliches (1971)

"The main advantage of a chain index is in the reduction of errors of approximation as the economy moves from one production configuration to another. If weights could be changed continuously, errors of this type would be eliminated. This property of Divisia indexes … characterizes no other index number. Discrete chain-linked index numbers reduce errors of approximation to a minimum. For this reason chain indexes rather than a single base period should be used in real product accounting and productivity measurement."

Two approaches

1. Try to find the best discrete approximation to an ideal Divisia index (Jorgenson and Griliches)

2. Ignore Divisia. Find the best discrete index number using *either* economic theory *or* the test approach *or* purely pragmatic criteria.

The second approach seems to be preferred by most NSIs who tend to be pragmatic. But this leaves chain-linking (which flows naturally from the Divisia approach) up in the air.

Divisia indices
François Divisia (1889-1964)

Divisia indices are …

- Consistent with production theory (Hulten's Theorem)
- Value-consistent (P x Q = value index)
- Aggregation-consistent

But Divisia indices are defined in continuous time so have to be approximated by discrete indices.

Divisia indices

$$\hat{Q}^D(0,t) := \sum_{i=1}^{i=N} w_i(t)\hat{q}_i(t)$$

$$\hat{P}^D(0,t) := \sum_{i=1}^{i=N} w_i(t)\hat{p}_i(t)$$

$w_i(t)$: value share of i at time t

Divisia (1925-26); Hulten (1973); Griliches and Jorgenson (1967)

Desirable properties (1)

Divisia indices are consistent with the theory of production.

Aggregate TFP growth = Growth of Divisia index of GDP *minus* Divisia index of aggregate primary input
= Domar-weighted sum of industry-level TFP growth rates

assuming the economy is efficient (P = MC):

$$\frac{d\ln TFP}{dt} = \frac{d\ln Y}{dt} - \frac{d\ln J}{dt}$$
$$= \sum_{i=1}^{N} \left(\frac{p_i Z_i}{GDP} \right) \frac{d\ln TFP_i}{dt}$$

(Hulten's Theorem)

Y : Divisia index of real GDP; J : Divisia index of real primary input; Z: real gross output

Domar (1961); Hulten (1978); Gabaix (2011); Baqaee and Farhi (2019)

Desirable properties (2)

- Value consistency (*P* x *Q* = value index):

$$P^D(0,t)Q^D(0,t) = \frac{\sum_{i=1}^{i=N} p_i(t)q_i(t)}{\sum_{i=1}^{i=N} p_i(0)q_i(0)}$$

- Aggregation consistency. Divisia index of GDP over all industries = Divisia index of Divisia indices of sub-aggregates like Manufacturing and Services.

But Divisia indices are defined in continuous time so have to be approximated by discrete indices.

Superlative index numbers

Superlative index numbers

Definition

A superlative index number is one which is exact for a *flexible functional form*

A flexible functional form is one which approximates to second order any linear homogeneous function acceptable to economic theory

Diewert (1976) and (1978)

Quadratic mean of order *r*

Flexible functional form (quantity case):

$$f_r(q) = \left[\sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij} q_i^{r/2} q_j^{r/2} \right]^{1/r} , \; a_{ij} = a_{ji}, \; \forall i \neq j , \; r \neq 0$$

Superlative quantity index number which is exact for this form is:

$$Q_r^{s,t}(p^s, p^t; q^s, q^t) = \left[\frac{\sum_{i=1}^{N} (q_i^t / q_i^s)^{r/2} (p_i^s q_i^s / p^s \cdot q^s)}{\sum_{k=1}^{N} (q_k^s / q_k^t)^{r/2} (p_k^t q_k^t / p^t \cdot q^t)} \right]^{1/r} , \; r \neq 0$$

Two special values of *r*

r = 0

By taking the limit as $r \to 0$ we find:

- The flexible functional form is *translog*
- The corresponding superlative index number is *Törnqvist*
- The Törnqvist is neither value-consistent not aggregation-consistent

r = 2

The superlative index number is *Fisher* (as used in US NIPAs)
The Fisher is value-consistent but not aggregation-consistent

Two-year versus chained indices

- Growth between year 0 and year T could be measured by *either* a chained superlative index, i.e. using the weights of years 0, 1, 2, … , T, *or* a two-year superlative index, i.e. using the weights of 0 and T.

- If economic behaviour is correctly described by a quadratic mean of order r, the results are identical (Diewert 1976). But suppose in practice they are not identical?

- Intuitively, chained indices seem superior to fixed base (Lowe) indices. But why are chained indices superior to two-year indices?

- Many economists seem to think that superlative index numbers solve the index number problem. They all approximate each other to 2nd order so any differences between them must be slight. Right?
- But Hill (2006) shows otherwise. Differences can be large, even lying outside the Laspeyres-Paasche spread.
- Hill (2006) used 2-year indices. Do his results still hold for chained indices?
- Time to look at this again using the *BEA/BLS integrated industry-level production account.*

The BEA/BLS integrated industry-level production account, 1987-2020

- Data constructed in accordance with the KLEMS methodology.

  Jorgenson et al. (1987)

- Annual gross output, value added, intermediate input, capital input, labour input (all in both nominal and real terms), and TFP
- 63 industries covering the whole economy including government
- 1987-2020

Garner et al. (2018), (2020) and (2021).

Empirical programme

- Calculate quadratic mean of order *r* indices of real GDP and the price of GDP from data for these 63 industries for a range of values of *r*:

  r = -20, -19, … , -2, -1, 0, 1, 2, … , 19, 20 (same as Hill (2006))

for the period 1987-2019 (also sub-periods 1987-2000 and 2000-2019).

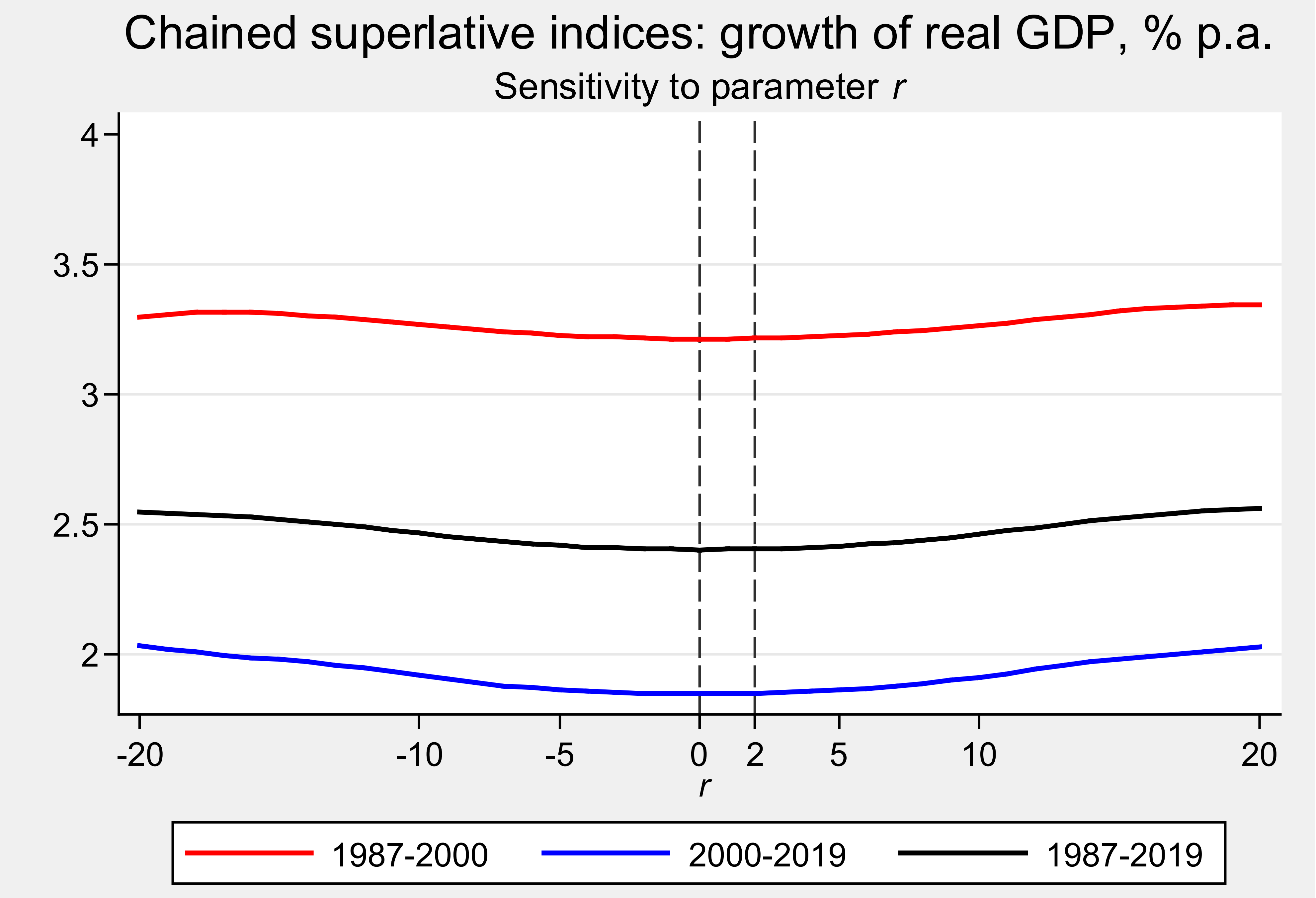
- Do the same for 9 industry groups (sub-aggregates) adding up in total to GDP.

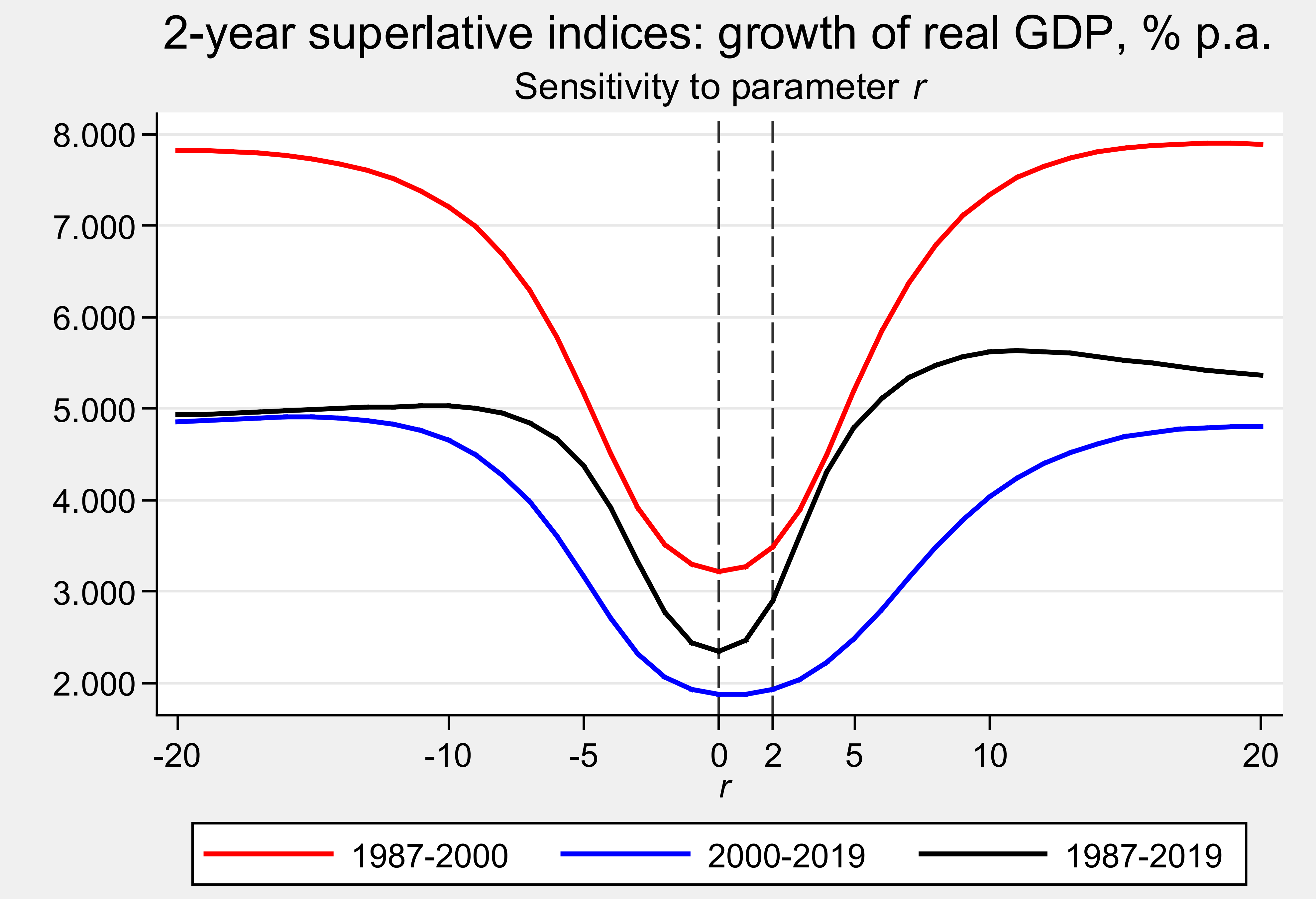
- Calculate both annually chain-linked and 2-year indices

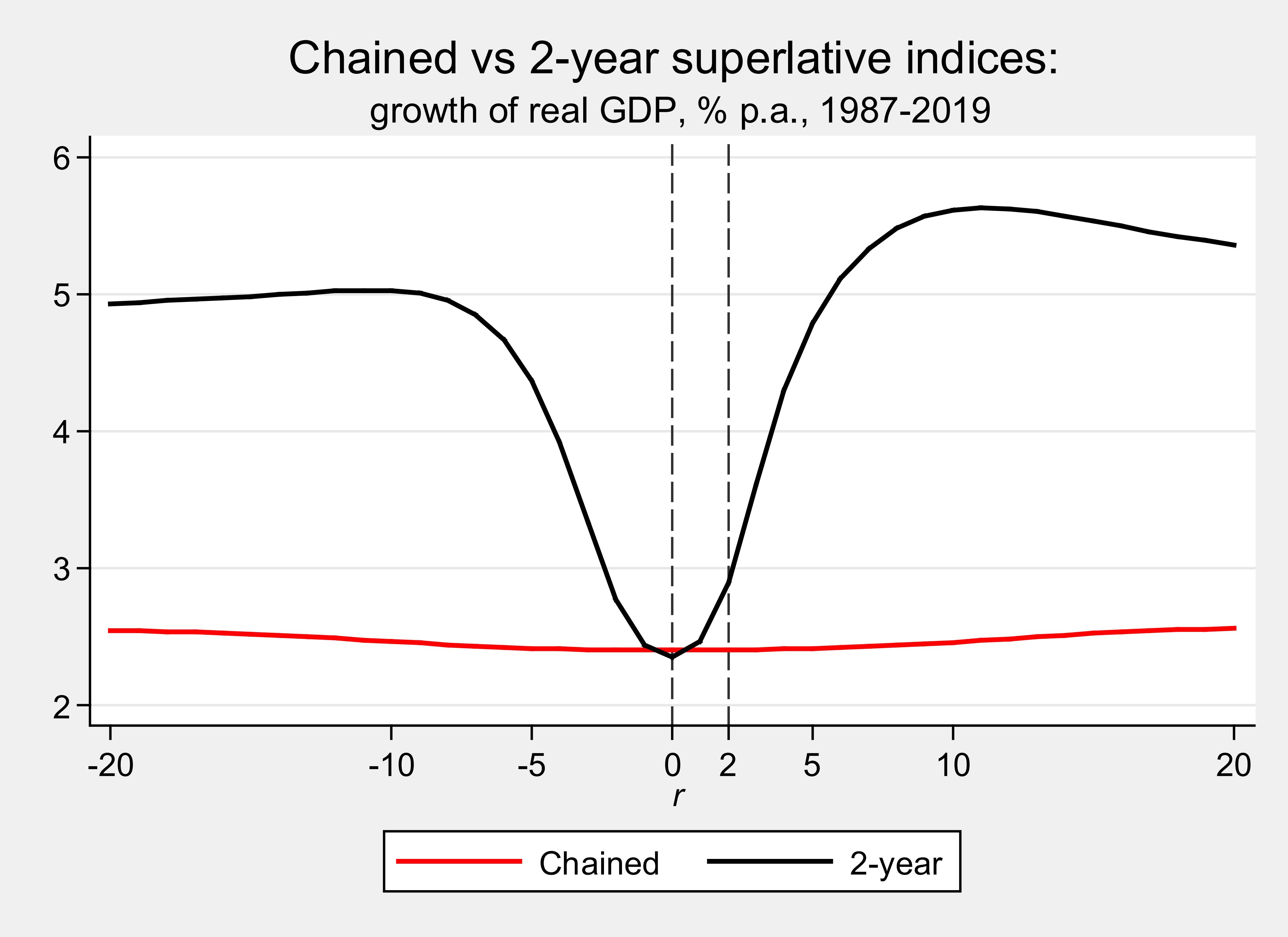
  (2-year indices use weights from the first and last years of the period; chain-linked indices use weights from all years in the period)

Questions to be answered

- How sensitive are the estimates to the value of r?
- How close to value consistency and aggregation consistency are the estimates for different values of r?
- How much difference does chaining make? I.e. how similar are the chained indices to the 2-year indices?


Chained superlative indices: growth of real GDP, % p.a.
Sensitivity to parameter r
4
3.5
3
2.5
2
-20
-10
-5
0
2
5
10
20
r
1987-2000
2000-2019
1987-2019


2-year superlative indices: growth of real GDP, % p.a.
Sensitivity to parameter r
8.000
7.000
6.000
5.000
4.000
3.000
2.000
-20
-10
-5
0
2
5
10
20
r
1987-2000
2000-2019
1987-2019


Chained vs 2-year superlative indices:
growth of real GDP, % p.a., 1987-2019
6
5
4
3
2
-20
-10
-5
0
2
5
10
20
r
Chained
2-year

Growth rates of real GDP in US, 1987-2019. % p.a.

Chained Törnqvist	2.402
Chained Fisher (US method)	2.404
Chained Laspeyres (European method)	2.481
2-year Törnqvist	2.350
2-year Fisher	2.893
“2-year” Laspeyres (i.e. Lowe, 1987 base)	3.757

Conclusions

- The findings of Hill (2006) are confirmed for 2-year superlative indices: they can be quite sensitive to the value of *r.*
- But *chained* superlative indices are much less sensitive, even for "extreme" values of *r*. So chaining makes a big difference.
- Also, chained superlative indices are both value-consistent and aggregation-consistent to a high degree of approximation.
- Chained Laspeyres (the European method) produces very similar results to the chained Fisher (US method).
- Chaining is a natural consequence of the Divisia approach.