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The Divisia approach to measuring output and productivity: with an application to the BEA-BLS integrated industry-level production account, 1987-2020

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CONTENT WARNING

Contains neoclassical economics. Despite every care being taken in the writing of this paper, trace elements of racism, colonialism, sexism, homophobia, islamophobia, transphobia, ableism, and classism may be present. The author is a member of the Centre for Macroeconomics at the London School of Economics. LSE's current Director has declared LSE to be a systemically racist institution. The author's thinking and research has not yet been fully decolonised under LSE's Race Equity Framework. (https://info.lse.ac.uk/staff/divisions/equity-diversity-and-inclusion/EDI-at-LSE/Race-Equity-at-LSE/race-equity-framework/Race-Equity-Framework).

Abstract

This paper analyses and illustrates the Divisia approach to measuring output and productivity. It argues that Divisia index numbers are the ideal to which real world index numbers should aspire, as argued by Jorgenson and Griliches (1971). Divisia index numbers are consistent with production theory and have a number of desirable properties, principally value consistency and aggregation consistency. But they are defined in continuous time and so must be approximated in practice by discrete index numbers, such as the traditional Laspeyres or Paasche or one of the superlative index numbers introduced by Diewert (1976). The alternative approach is to ignore Divisia and start with discrete index numbers. The issues involved here are illustrated by examining data from the BEA/BLS industry-level integrated production account, 1987-2020. Estimates of superlative and other index numbers are presented for this dataset. The sensitivity of real GDP growth to the value of the crucial parameter in a superlative index number is tested. The extent to which value consistency and aggregation consistency are satisfied for different superlative index numbers are analysed. Chaining is a natural consequence of the Divisia approach but does not follow so automatically from the use of superlative indices. So I also compare chained and unchained versions of these same index numbers. Finally, Europe uses a different approach to output measurement to the US, chained Laspeyres versus chained Fisher. I look at how different US estimates would be if they employed European methodology.

1. Introduction[1]

This paper analyses and illustrates the Divisia approach to measuring output and productivity. It argues that Divisia index numbers are the ideal to which real world index numbers should aspire. Divisia index numbers are consistent with production theory and have a number of desirable properties, principally value consistency and aggregation consistency. But they are defined in continuous time and so must be approximated in practice by discrete index numbers, such as the traditional Laspeyres or Paasche or one of the superlative index numbers introduced by Diewert (1976). The alternative approach is to ignore Divisia and start with discrete index numbers. The issues involved here are illustrated by examining data from the BEA/BLS industry-level integrated production account, 1987-2020. Estimates of superlative and other index numbers are presented for this dataset.

The approach I employ here was previously advocated by Jorgenson and Griliches (1971) in a response to a critic of their classic paper on US productivity growth (Griliches and Jorgenson 1967). They state:

> "The main advantage of a chain index is in the reduction of errors of approximation as the economy moves from one production configuration to another. If weights could be changed continuously, errors of this type would be eliminated. This property of Divisia indexes, called "invariance" by Richter, characterizes no other index number. Discrete chain-linked index numbers reduce errors of approximation to a minimum. For this reason chain indexes rather than a single base period should be used in real product accounting and productivity measurement."

Discrete versus continuous index numbers

In practice Divisia indices cannot be calculated exactly since data are only available at discrete intervals rather than continuously. But they can be approximated by chained indices of which the most commonly used for volume changes are the annually chained Laspeyres, Fisher or Törnqvist. Economic modellers and productivity analysts (following Griliches and Jorgenson 1967 and Jorgenson et al. 1987) often use the Törnqvist. National income accountants generally use either the chained Laspeyres (mandated by Eurostat (2013) for EU countries) and also still used by the UK, or the chained Fisher (as in Canada and the US). The

[1] This paper is enormously indebted to the work of the late Dale Jorgenson extending over decades. Thanks to Jon Samuels for some advice and assistance with the BEA/BLS dataset.

US shifted its National Income and Product Accounts (NIPA) to a chain-linked Fisher index from a fixed base one in 1996. But in Europe (including the UK) it has been customary to update the weights every five years or so. In other words, most of Europe has always used a form of chain-linking (France was an exception in using a fixed base). In the UK case the change from what might be called quinquennial chain-linking to annual chain-linking took place in 2003.

The 2008 SNA has a whole chapter (Chapter 15) devoted to price and volume measures (European Commission et al. 2009). Unfortunately nowhere does it mention Divisia index numbers. Despite this I am arguing that real world price and volume indices are best thought of as (more or less good) approximations to the ideal, the Divisia index. This approach enables us to link economic theory to the practice of national income accounting without having to assume particular functional forms for the underlying relationships like utility functions or production functions. The Divisia approach enables one to prove intuitively plausible propositions which one would otherwise struggle to establish (Oulton 2021). Large changes can be handled as well as small ones.

The alternative approach is to assume that economic behaviour can be explained exactly by utility or production functions which take the form of a "quadratic mean of order r". These functional forms are second order approximations to any functions acceptable to economic theory. Then there is a superlative index number (dependent on the parameter r) which is exact for this particular functional form (Diewert 1976; Mizobuchi and Zelenyuk 2021). Furthermore this index number measures large changes correctly as well as small ones. The drawbacks to this approach are that the results are dependent on the choice of the parameter r, and that the attractive properties of the Divisia index − price index times volume index equals value index and consistency in aggregation – are either lost, or compel the choice of a particularly value for r. For example setting $r = 2$ results in the Fisher index which satisfies the first of these properties but not the second, consistency in aggregation. Setting $r = 0$ results in the Törnqvist index which satisfies neither property. I am not aware of any superlative index number which satisfies both properties.

At first sight a discrete approach may seem more realistic in economics. But this is not the case. To be sure, agents do not make decisions in continuous time; if for no other reason, they take time off for sleep, weekends and holidays. But neither do they take decisions in accordance with the usual discrete time formulation. Superlative index number theory assumes optimization. This means that all decisions (what to buy, what to sell) are assumed to be made either at the beginning or at the end of the period. But how long is the period? In practice this is nowadays either a calendar year or a quarter. But the quarterly and annual models are not the same. Which is chosen depends on the availability of data. In reality the interval between successive optimizations probably depends on the nature of the decision, with decisions about a firm's investment for example being made less frequently than decisions about hours worked. Either way, some at least of the observed data, for example sales, are likely to be time averages over the chosen period (whether a year or a quarter). Price data is usually collected monthly so annual or quarterly data are averages of monthly point-in-time observations. The consequences of all this have not been incorporated into the theory.

There is an interesting contrast here between economics and the natural sciences. Since the early nineteenth century physicists have been studying the flow of heat in material bodies. In the standard heat equation both time and matter are taken to be continuous. But physicists have accepted for well over a century that matter is made up of discrete objects called molecules, in turn composed of atoms, also discrete objects. (Physicists are also aware of the possibility that time too may be fundamentally discrete or quantized but as yet no conclusive evidence has been found for this). So the heat equation is at variance with the known facts of the world. This has not stopped physicists using it since, at the macroscopic scale where the equation is applied, the continuous assumption has yet to produce predictions at variance with observation. Because analytical solutions of the heat equation are not always available, physicists often solve it using numerical methods which necessarily require making both time and matter discrete. So now we have a discrete approximation to a continuous model which is in turn an approximation to a (different) discrete reality. The approach advocated here is analogous to that of the physicists.

The two approaches

Many economists, noting the results of Diewert (1976) and others that all superlative index numbers are second order approximations to each other and (on certain assumptions) to the unknown function generating the data, have concluded that the "index number problem" has been solved. However Hill (2006), in an article provocatively titled "Superlative index numbers: not all of them are super", has used US data to show that the estimated growth rates are in practice quite sensitive to the value chosen for the parameter *r*. Since there seems no a priori reason for preferring one value of *r* over another, and in the absence of any empirical evidence on the true value of *r*, the issue still seems to be up in the air. Hill (2006) used what I call here two-year indices (see Section 2 below), not chained indices, so an important issue still to be resolved is whether chaining affects his conclusions.

Plan of the paper

Section 2 reviews the theory of Divisia index numbers and of superlative index numbers. Section 3 briefly describes the dataset to be used for the empirical testing, the BEA/BLS integrated industry-level production account. Then section 4 presents the results. First of all the sensitivity of real GDP growth to the value of the crucial parameter *r* in a superlative index number is tested. Then the extent to which value consistency and aggregation consistency are satisfied for different superlative index numbers is analysed. Chaining is a natural consequence of the Divisia approach but does not follow so automatically from the use of superlative indices. So I also compare chained and unchained versions of these same index numbers. Finally, Europe uses a different approach to output measurement to the US, chained Laspeyres versus chained Fisher. I look at how different US estimates would be if they employed European methodology. Section 5 summarises the conclusions and also points to other issues in need of exploration.

2. Theory

Divisia indices[2]

Divisia indices have one overarching advantage: they are consistent with production theory. Let $Y = (Y_1, \ldots, Y_N)$ denote the vector of final demands and $J = (J_1, \ldots, J_K)$ the vector of primary factor supplies. Then consider an economy whose social production possibility frontier can be represented by

$$F(Y, J, t) = 0$$

[2] Divisia indices were devised by Divisia (1925-1926). They were introduced explicitly into productivity analysis by Griliches and Jorgenson (1967). They have been analysed by Richter (1966), Hulten (1973) and Balk (2005). Earlier researchers, e.g. Solow (1957), had used discrete chained indices to measure aggregate input.

Assuming perfect competition and constant returns to scale, Hulten (1978) showed that the rate at which the social production possibility frontier is shifting out over time (the aggregate TFP growth rate) can be measured by the difference between the growth rate of a Divisia index of final demands and the growth rate of a Divisia index of primary factor supplies. (Since aggregate final demand equals aggregate value added, the Divisia index of final demands equals the Divisia index of value added.) Furthermore this rate is identical to a weighted sum of industry-level TFP growth rates, the latter being derived from industry-level production functions. The industry-level TFP growth rate is the growth rate of industry output minus the growth rate of a Divisia index of intermediate inputs and minus the growth rate of a Divisia index of primary inputs into that industry. In symbols:

$$\frac{d\ln TFP}{dt} = \sum_{i=1}^{N} \left(\frac{p_i Z_i}{GDP} \right) \frac{d\ln TFP_i}{dt} \tag{1}$$

where Z_i is the gross output of the ith industry, p_i is its price, and *GDP* is nominal GDP. The weight on each industry is its sales divided by nominal GDP, the so-called Domar weight first suggested by Domar (1961).

This result, the equality of the two ways of calculating aggregate TFP growth, is called Hulten's Theorem by Gabaix (2011), and is proved by the latter in more general terms (see his Appendix B, also Baqaee and Farhi 2019). The point to note here is that whichever way we choose to calculate aggregate TFP growth, we will need to calculate Divisia indices of outputs and inputs. [3]

Divisia indices also possess two further desirable properties. The first property is *value consistency*: the product of a Divisia quantity index and a Divisia price index is the expenditure ratio (total value of all commodities or outputs in the second period divided by the total value in the first). The second property is *aggregation consistency*.

Value consistency arises from the definition of Divisia index numbers. Consider the value (V) of some aggregate at time t relative to its value in some reference period, say time 0:

[3] Footnote 45 of Baqaee and Farhi (2019) compares their second-order approximation approach to measuring TFP shocks to the Divisia one (though without using his name). They argue that the two approaches are essentially equivalent. They argue that their approach, which uses calibration to estimate some key parameters, is necessary for making predictions while the Divisia one can only be used to analyze the past.

$$\frac{V(t)}{V(0)} \equiv \frac{\sum_{i=1}^{i=N} p_i(t)q_i(t)}{\sum_{i=1}^{i=N} p_i(0)q_i(0)} \tag{2}$$

where $p_i(q_i)$ is the price (quantity) of the ith item and all variables are assumed to be differentiable functions of time (*t*). We want to split up the right hand side of (2) into a price index and a quantity index:

$$\frac{V(t)}{V(0)} \equiv P^D(0,t)Q^D(0,t) \tag{3}$$

Here $P^D(0,t)$ is the Divisia price index and $Q^D(0,t)$ is the Divisia quantity index for period *t* relative to period 0. Taking logs and totally differentiating equation (2) with respect to time:

$$\hat{V}(t) = \sum_{i=1}^{i=N} w_i(t)\hat{p}_i + \sum_{i=1}^{i=N} w_i(t)\hat{q}_i \tag{4}$$

Here a hat (^) indicates a logarithmic growth rate, i.e. $\hat{X} := d \ln X / dt$, and

$$w_i(t) := \frac{p_i(t)q_i(t)}{V(t)}, \quad i = 1,\ldots,N$$

are the value shares.[4] Now identify the first summation on the right hand side of (4) with the Divisia price index and the second with the Divisia quantity index:

$$\hat{P}^D(0,t) := \sum_{i=1}^{i=N} w_i(t)\hat{p}_i(t) \tag{5}$$

and

$$\hat{Q}^D(0,t) := \sum_{i=1}^{i=N} w_i(t)\hat{q}_i(t) \tag{6}$$

From (4)

$$\hat{V}(t) = \hat{P}^D(0,t) + \hat{Q}^D(0,t) \tag{7}$$

Consistent with (3), we normalize by setting the price index and the quantity index equal to 1 in the reference period 0: $P^D(0,0) = Q^D(0,0) = 1$. Since the growth rate of the value index equals the sum of the growth rates of the price and quantity indices and since the level of the value index satisfies value consistency when *t* = 0, it follows that value consistency must hold in all time periods.

The levels of the price index and the quantity index in any time period *T* can be now found by integration:

$$\ln P^D(0,T) = \int_0^T \sum_{i=1}^{i=N} w_i(t)\hat{p}_i(t)dt \tag{8}$$

[4] Here and below the symbol ":=" should be read as "is defined to be equal to".

$$\ln Q^D(0,T) = \int_0^T \sum_{i=1}^{i=N} w_i(t)\hat{q}_i(t)dt \tag{9}$$

The solutions for the price and quantity indices are line integrals and it has long been known that these are not in general path-independent (in the economics literature, see Hulten 1973). That is to say, the level of the index at *T* relative to 0 depends not just on the prices and quantities prevailing in periods 0 and *T* but also on all the prices and quantities at intermediate points on the interval (0, *T*). So different paths between 0 and *T* may produce different outcomes at *T* even though prices and quantities at 0 and *T* are by assumption the same for all paths. But taking the economic approach to index numbers, it has also been established that in a consumer context, if economic behaviour is rationalizable by a homothetic utility function, then a Divisia index of consumer prices is path-independent. In a producer context, if there are constant returns to scale, then again a Divisia quantity index of output is path-independent. The economic approach assumes optimizing behaviour by agents.[5]

Now consider the second desirable property, aggregation consistency. Aggregation consistency means that we can (in principle) calculate a Divisia index in two ways. Either we can calculate it directly from the basic elements at the lowest level (e.g. commodities or industries), the one-step method. Or we can calculate it in two (or more) steps: first calculate an index for each sub-aggregate of interest and then calculate the overall index of these sub-aggregate indices, the multi-step method. Either method will produce the same answer at the aggregate level (e.g. GDP). For example, suppose we have time-series data on real value added for *N* industries. We can calculate a Divisia index of real GDP in one step from data on real value added for these *N* industries. Or we can divide the *N* industries up into say Manufacturing and Services and define Divisia indices for Manufacturing output and Services output. Then we can define a two-step index for GDP as a Divisia index of the two Divisia indices for Manufacturing and for Services. The second method will produce the

[5] See Apostol (1957) for the mathematical background on path-dependence and Hulten (1973) for the economic interpretation. Homotheticity of the utility function is not a very attractive assumption given the overwhelming evidence for Engel's Law. Oulton (2008) for time-series data and Oulton (2012) for cross-country data (see also Oulton (2015)) develop practical ways to estimate modified Divisia indices (Konüs indices) which hold utility constant at some specified level and are also path-independent. These methods could also be applied in the producer context if returns to scale are not constant.

same result for GDP as the first. Specifically, for this two-sector example the one-step quantity index is

$$\hat{Q}^D_{GDP}(t) = \sum_{i=1}^{i=N} w_i(t)\hat{q}_i(t) \tag{10}$$

Assuming that Manufacturing comprises the first M industries and Services the remainder (industries $M + 1$ to N), the quantity indices for the two sub-aggregates are

$$\hat{Q}^D_{Manu}(t) = \sum_{i=1}^{i=M} w_{iM}(t)\hat{q}_i(t)$$

and

$$\hat{Q}^D_{Serv}(t) = \sum_{j=M+1}^{j=N} w_{jS}(t)\hat{q}_j(t)$$

where the value shares in Manufacturing and Services respectively are defined as

$$w_{iM}(t) := \frac{p_i(t)q_i(t)}{\sum_{i=1}^{i=M} p_i(t)q_i(t)}$$

and

$$w_{jS}(t) := \frac{p_j(t)q_j(t)}{\sum_{j=M+1}^{j=N} p_j(t)q_j(t)}$$

(Here the prices and quantities should be understood as those of industry-level value added). The growth of the two-step index of real GDP is

$$\hat{Q}^D_{GDP,2-step}(t) = w_M(t)\hat{Q}^D_{Manu}(t) + w_S(t)\hat{Q}^D_{Serv}(t)$$

where the value shares of the two sectors in GDP are

$$w_M(t) := \frac{\sum_{i=1}^{i=M} p_i(t)q_i(t)}{\sum_{i=1}^{i=N} p_i(t)q_i(t)}$$

and

$$w_S(t) := \frac{\sum_{j=M+1}^{j=N} p_j(t)q_j(t)}{\sum_{i=1}^{i=N} p_i(t)q_i(t)}$$

Using the definitions of the value shares, it is now straightforward to verify that

$$\hat{Q}^D_{GDP,2-step}(t) = \hat{Q}^D_{GDP}(t)$$

Clearly this argument carries over to price indices and also generalizes to any number of sectors. This proves consistency in aggregation.

Superlative index numbers

Diewert (1976) defined a superlative index number as one which is exact for a flexible functional form. In turn a flexible functional form is one which approximates to second order any linear homogeneous function acceptable to economic theory. A second order approximation is one where the approximating function and the other function are equal as are their first and second derivatives, *at some common point*. He developed a family of flexible functional forms called quadratic means of order *r*. In the quantity case this takes the form:

$$f_r(q) = \left[\sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij} q_i^{r/2} q_j^{r/2} \right]^{1/r}, \; a_{ij} = a_{ji}, \; \forall i \neq j, \; r \neq 0 \tag{11}$$

(The case $r = 0$ is handled by taking the limit as r goes to zero which yields the translog aggregator function.) Equation (11) can be interpreted as a production possibly frontier with primary input supplies and technology held constant. It arises out of an economy in which producers act as if they were maximising profits under constant returns to scale with output and input prices taken as given. (There is an analogous form for the corresponding price or unit cost possibility frontier.) The corresponding (superlative) quantity index for period t relative to period s is

$$Q_r^{s,t}(p^s, p^t; q^s, q^t) = \left[\frac{\sum_{i=1}^{N} (q_i^t / q_i^s)^{r/2} (p_i^s q_i^s / p^s \cdot q^s)}{\sum_{k=1}^{N} (q_k^s / q_k^t)^{r/2} (p_k^t q_k^t / p^t \cdot q^t)} \right]^{1/r}, \; r \neq 0$$

$$= \prod_{i=1}^{N} \left(\frac{q_i^t}{q_i^s} \right)^{\frac{1}{2}\left[(p_i^s q_i^s / p^s \cdot q^s) + (p_k^t q_k^t / p^t \cdot q^t) \right]}, \qquad r = 0 \tag{12}$$

(The case $r = 0$ can also be thought of as the limit of the first line on the right hand side as r approaches 0.) Here superscripts denote discrete time periods and $p^t (q^t)$ is the price (quantity) vector in period t; the numerator contains the budget shares in the earlier period s, and the denominator contains the shares in the later period t. This index is exact for the flexible functional form (11).[6]

A number of special cases of (11) and (12) are of interest. First, when $r = 0$ the function takes the translog form and the corresponding index number takes the Törnqvist form.

[6] See now also Mizobuchi and Zelenyuk (2021) for more on quadratic mean of order *r* indices.

Second, when $r = 2$ the index takes the Fisher "ideal" form (the geometric mean of the Paasche and Laspeyres indices). Third, when the off-diagonal coefficients in (11) are all zero, the function takes the CES form but this is now not a flexible functional form.

Hill (2006) proved an important property of superlative index numbers: The limit of the quantity (price) index number as $|r| \to \infty$ is the geometric mean of the largest and smallest quantity (price) relatives:

$$\lim_{r\to+\infty} Q_r^{s,t} = \lim_{r\to-\infty} Q_r^{s,t} = \left[\min\left(\frac{q_i^t}{q_i^s}\right) \max\left(\frac{q_i^t}{q_i^s}\right) \right]^{\frac{1}{2}} \tag{13}$$

An analogous expression holds for price indices.

Superlative index numbers do not have the consistency-in-aggregation property. However Diewert (1978) showed that the class of superlative index number formulae has an approximate consistency-in-aggregation property.

When considering discrete index numbers it is also worth noting that the Laspeyres-Paasche pair (either a Laspeyres quantity with a Paasche price index or a Laspeyres price with a Paasche quantity index) possess both value and aggregation consistency. Value consistency means that

$$P_{Paas}^{s,t} Q_{Lasp}^{s,t} = P_{Lasp}^{s,t} Q_{Paas}^{s,t} = \frac{\sum_{i=1}^{N} p_{it} q_{it}}{\sum_{i=1}^{N} p_{is} q_{is}} = \frac{V(t)}{V(s)} \tag{14}$$

which follows from the definitions of the Laspeyres and Paasche indices. Aggregation consistency means that a two-step Laspeyres (Paasche) index is exactly equal to a one-step Laspeyres (Paasche) index. This property also follows easily from the definitions. But neither Laspeyres nor Paasche are superlative indices.

Chaining and discrete index numbers

Equations (5) and (6) make clear that Divisia indices use a continuous form of chaining: the weights shift continuously over the period studied. Developed countries have gradually shifted towards calculating their own discrete price and quantity index numbers in chained form; e.g. EU countries and the UK use chained Laspeyres while the US uses chained Fisher indices. Economists who prefer an axiomatic to an economic approach to index numbers and are certainly not willing to place Divisia indices at the foundation of economic measurement (e.g. Balk 2008 and 2010) are agnostic about the virtues of chaining. And as we have seen official statisticians do not base their case for chaining on the Divisia approach.[7] So other than this, what is the justification for chaining, beyond a rather vague desire "not to let the weights get too out of date"? This last argument certainly fails in the case of two-period symmetric indices like the Fisher which give equal importance to the first and last period weights.

Diewert (1976) proved an important result about chained and non-chained superlative indices when these take the form of a quadratic mean of order *r*. Consider the growth of quantities over the period (0, 2). From (11) we have

$$\frac{f_r^2}{f_r^0} = \frac{f_r^1}{f_r^0}\frac{f_r^2}{f_r^1} \tag{15}$$

Because the index numbers Q_r are exact for the functional form (11), it follows from (15) that

$$Q_r^{0,2} = Q_r^{0,1} \times Q_r^{1,2} \tag{16}$$

In other words the two-period index number and the chained one produce the same result numerically, provided of course that economic agents are indeed maximizing (11).[8] In other words, superlative index numbers satisfy the circularity test. This result obviously generalizes to many time periods:

$$Q_r^{0,T} = Q_r^{0,1} \times Q_r^{1,2} \times \ldots \times Q_r^{T-2,T-1} \times Q_r^{T-1,T} \tag{17}$$

Diewert (1976) argued that this fact allowed an empirical test of the theory. If in practice the circularity test is failed then either producers are not maximizing or they are maximizing something other than $f_r(q)$. Hence he recommended chaining since the aggregator function is unlikely to remain constant over long periods of time. This is because the slope of the production possibility frontier, at points where the ratios of quantities produced are equal (along a ray from the origin), will change over time if technical progress occurs at different

[7] It seems likely that the US adopted annual chain-linking at least in part because of the disruptive effects of the rapidly falling price of computers on the national accounts. This meant that moving from an earlier fixed base to a later one caused all previously published GDP growth rates to be revised downwards (in the absence of any data revisions), which was embarrassing to the Clinton administration; see *The Teaching Economist*, Issue 11, Spring 1996.

[8] Note that just substituting into (16) from the index number formula (12) does not prove the result. One requires also maximizing behaviour on the part of producers.

rates in different industries or if supplies of primary inputs are not all growing at the same rate. In other words the a_{ij} parameters in (11) will in general be changing over time. Only if the production possibility frontier at time *t* is just a radial blow up of its position at any previous point will its slope at a given ratio of outputs be unchanged. That is, if the aggregator function now depends on time, now written $f_r(q,t)$, we require that

$$f_r(q,t) = g(t) f_r(q,0) \tag{18}$$

This is obviously a highly restrictive assumption since it implies that the supplies of each primary input are rising at the same rate and that TFP is growing at the same rate in every industry.

The non-chained quantity index between any two time periods *s* and *t* for a quadratic mean of order *r* is defined by equation (12) but we now write the left hand side more compactly as $Q_r(s,t)$. The chained index over the same time period is now written as

$$Q_r^{Ch}(s,t) = Q_r(s,s+1) \times Q_r(s+1,s+2) \times \ldots \times Q_r(t-2,t-1) \times Q_r(t-1,t) \tag{19}$$

It is then an empirical issue as to how similar are the chained $(Q_r^{Ch}(s,t))$ and the non-chained two-period quantity indices $(Q_r(s,t))$ over a given time period (*s, t*).

One advantage of chaining when using superlative indices is that adding an additional time period doesn't require us to change the past (in the absence of data revisions). Under chaining the growth rate over the interval (*s*, *t*) is unchanged when we extend the overall period to *t* + 1; we just add another link in the chain for the last period. But without chaining the growth rate over (*s*, *t*) becomes problematic. Should we continue to measure it using the weights of just *s* and *t*, while using those of *s* and *t* + 1 to measure growth over the whole interval (*s*, *t* + 1)? If so, then how should we measure growth from *t* to *t* + 1? We have two choices. Either we can use the weights of *t* and *t* + 1, i.e. $Q_r(t,+1)$, or we can use the growth rate implied by growth over the two long intervals (*s*, *t*) and (*s*, *t* + 1), i.e. $Q_r(s,t+1)/Q_r(s,t)$. The answers will not be the same unless (18) is satisfied.[9]

[9] Another possibility is to use the GEKS index commonly applied to cross-country or cross-regional data. This approach takes a geometric mean of indices over all possible paths between year *s* and year *t*, including the direct one¸ which ensures transitivity. However this suffers from the drawback that all the original growth rates change when an additional year is added, just as in a cross-section context the relative levels change when an additional country or region is added.

US versus European methodology

As stated above, the US NIPAs use a chained Fisher for measuring real GDP growth while EU countries and the UK use a chained Laspeyres. The chained Laspeyres has some advantages over the chained Fisher since it is both value consistent (when paired with a chained Paasche index) and aggregation consistent. However in Europe price indices such as the CPI are usually chained Laspeyres not chained Paasche. A more serious drawback, one known since at least Bruno and Sachs (1985), is that a Laspeyres index predicts (wrongly) that an exogenous worsening in the terms of trade *increases* GDP.[10] Nor does chaining help since the error remains and will impact on the average growth rate over any interval which includes the period when the terms of trade worsened. So even changes in the terms of trade which are reversed over time will lead to a systematic overprediction of GDP growth. Nonetheless it is still of interest to see how much difference it would make had the US adopted European methodology.

3. Data: the BEA-BLS industry-level production account

The idea now is to use actual data to test the extent to which real world indices are consistent with the desirable properties of Divisia indices. For this purpose I employ data from the BEA-BLS industry-level production account. The advantage of the BEA/BLS dataset for index number and productivity research is that it is highly consistent with production theory and based on a massive and detailed data-gathering exercise extending over many years.

The data are constructed in accordance with the KLEMS methodology pioneered by Jorgenson and his various collaborators: Jorgenson et al. (1987), (2005), (2016) and (2018). They give annual gross output, value added, intermediate input, capital input, labour input (all in both nominal and real terms), and TFP for 63 industries, classified by NAICS, covering the whole economy (including federal, state and local government). The period

[10] The correct answer is that if the import whose price has risen is an intermediate input like energy, then in an efficient economy (price = marginal cost) there is no effect on GDP (abstracting from any effects on aggregate demand). If there is a positive margin of price over marginal cost, then GDP *falls* in response to a rise in the imported input's price (Oulton 2021).

covered is currently 1987-2020.[11] Nominal value added in these 63 industries adds up to nominal GDP.

Real value added is double deflated. The growth of labour input is the share-weighted growth of hours worked for approximately 170 different groups of workers cross-classified by sex, eight age groups, six education groups, and employment class (payrolled vs. self-employed). The growth of capital input is the share-weighted growth rate of capital services based on about 100 types of capital including inventories and land. A full description of the BEA-BLS-industry-level production account is in Garner et al. (2020) and (2021). Further detail on methodology is available from Garner et al. (2018).

The data for 1987-2020 was downloaded from the BEA website (www.bea.gov) in the form of a spreadsheet named "BEA-BLS-industry-level-production-account-1987-2020.xlsx", available at https://www.bea.gov/data/special-topics/integrated-industry-level-production-account-klems. This spreadsheet was released on May 11 2022 and comprises the latest data available at the time this research was begun. It states: "This file contains the data underlying the BEA/BLS Integrated Industry-level Production Account for the United States. The data covers the 1987-2020 period and is updated to reflect the annual update to the input output accounts released on September 30, 2021 available here: https://apps.bea.gov/scb/2021/10-october/1021-industry-annual-update.htm".

4. Results

Before turning to the results, I start with a brief descriptive analysis of the dataset, focusing in particular on the extent of structural change between 1987 and 2019. In what follows I ignore 2020 as being too distorted by the pandemic to add any light. To give an idea of the importance of each industry and of structural change, Table 1(a) lists the industries, together with the share of each industry's value added in total value added (nominal GDP) in three years: 1987, 2000, and 2020. Industries vary widely in importance. In 2019 the smallest shares were for industries 21 (Apparel and leather and allied products) (0.04%), 31 (Water

[11] Extending the data back to 1947 would be highly desirable. At the moment however that cannot be done on a fully consistent basis. And the quality of the estimates for years prior to 1987 is lower (Eldridge et al. 2020).

transportation) (0.06%) and 20 (Textile mills and textile product mills) (0.07%), all in steep decline since 1987. The largest were industries 45 (Real estate) (11.13%) and 63 (State and local government) (11.74%), both rising since 1987.

Structural change, 1987-2019

Figure 1 shows the growth of prices in the 63 industries (measured as 100 x the log change in price) over 1987-2019. Much the largest fall in prices (almost 300%) occurred in a single industry: industry 13 (Computer and electronic products). The largest rise (181%) was in industry 44 (Funds, trusts, and other financial vehicles). Figure 2 shows the growth in quantities in the same 63 industries over 1987-2019. Quantity growth is more dispersed than price growth. The standard deviation of price growth was 62.8% while that of quantity growth was 81.3%.

The outcome of price and quantity growth is changes in shares. Figure 3 shows the changes in each industry's value added share between 1987 and 2019. 28 industries experienced positive growth in share and 35 negative growth over this period. The maximum change in share was +2.1 p.p. while the minimum was -2.1 p.p. The correlation coefficient between the shares in 1987 and in 2000 was 0.98 while that for the shares between 1987 and 2019 was 0.95. So on this measure, changes in shares, structural change was quite limited. The proximate reason for this modest change in shares is that price growth and quantity growth are negatively correlated: the correlation coefficient is minus 0.61 over 1987-2019. A negative correlation between price and quantity growth is often found empirically. It is relevant to a study of index numbers since it makes it likely that a Laspeyres (base-weighted) quantity index will grow more rapidly than a Paasche (current-weighted) index.

Though structural change appears quite modest at the industry level, a somewhat different picture emerges if the 63 industries are aggregated into 9 industry groups: see Table 1(b). The major changes apparent now over 1987-2019 are a fall of 7 percentage points in the Manufacturing share and a corresponding rise in the share of Other Services.

How sensitive is the chained index of real GDP growth to the choice of r?

We first test how sensitive the estimated growth rate of real GDP is to the choice of the parameter r (recall that the official estimates in the US NIPAs assume in effect that $r = 2$, the Fisher case). Estimates of the average annual growth rate of real GDP over the period 1987-

2019 according to the chained superlative index of equations (12) and (19) appear in Table 2, for a range of values of *r*. Here following Hill (2006) the parameter *r* is allowed to vary from -20 to + 20. This may seem an implausibly wide range, given that in practice a value of *r* of either 0 or 2 is usually employed. But as Hill (2006) points out, the size of *r* is an empirical matter and no one has in fact estimated *r* empirically.

It turns out (Table 2) that the estimated growth rates are symmetrical around a value of *r* at or close to zero. The estimated growth rates are not very sensitive to the value of *r*: taking *r* to be +20 yields a growth rate of 2.562% p.a. while the minimum is 2.402% p.a. when $r = 0$: see Figure 4 which also shows a similar picture for the sub-periods 1987-2000 and 2000-2019. On the other hands the volatility of the annual growth rate rises markedly as $|r|$ rises above about 10. From a practical point of view the estimated growth rate in the Törnqvist case $(r = 0)$ is almost identical to the Fisher case ($r = 2$): 2.402 versus 2.404 % p.a. If instead of chained Fisher the BEA employed the European method of chained Laspeyres, then growth over 1987-2019 would have been estimated as 2.481% p.a. rather than 2.404% p.a., a significant but relatively minor difference.

Assuming constant returns to scale suggests that the true estimate of the annual growth rate lies between the chained Laspeyres and the chained Paasche, i.e. in the range 2.327 to 2.481% p.a. Both the Fisher and the Törnqvist satisfy this criterion. On an annual basis the chained Fisher lies within the Laspeyres-Paasche spread in every year; the chained Törnqvist lies within the spread in all but two years, 2008 and 2020, both years of severe recession.

How closely is value consistency achieved when $r \neq 2$?

We know that value consistency is exactly achieved when $r = 2$, as in the Fisher index used officially in the US NIPAs. Here we test the extent of deviations from value consistency for a range of values of *r*. Specifically, we calculate an index of real GDP and an index of the price of GDP from the price and quantity data at the 63 industry level, using a range of values of *r*. The value consistency index is then defined as the ratio of the value index to the product of the price index and the quantity index (for a given value of *r*):

$$VC_r(0,t) := \frac{\sum_{i=1}^{N} p_{it} q_{it} / \sum_{i=1}^{N} p_{i0} q_{i0}}{P_r^{Ch}(0,t) Q_r^{Ch}(0,t)} \tag{20}$$

Note that the value consistency index is 1 in the reference year 0. Results for value consistency using chained indices appear in Table 3. For *r* lying between -5 and +5 consistency is high: the deviation from a value of 1 is less than about $\pm 1\%$. By contrast the chained Laspeyres shows a steadily increasing divergence from 1 over 1987-2019. By 2019 this index is only 0.9520.

Aggregation consistency

We now define the aggregation consistency index as the ratio of the level of the 2-step chained quantity index to the level of the 1-step chained quantity index. The first step of the 2-step index is chained quantity indices for each of 9 industry groups. The second step aggregates these to the GDP level. See Table 1(b) for the definitions of the 9 industry groups in terms of the underlying 63 industries. In symbols the aggregation consistency index is defined as

$$AC_r(0,t) := \frac{Q_r^{2,Ch}(0,t)}{Q_r^{Ch}(0,t)} \tag{21}$$

where $Q_r^{2,Ch}(0,t)$ is the 2-step index with parameter *r* in year *t* with reference year 0. Note that the index is 1 in the reference year: $AC_r(0,0) = 1$.

Table 5 shows the aggregation consistency index for selected values of *r*. For values of *r* in the interval (-5,+5) deviations from aggregation consistency are very small. Only outside that range do they become significant. For example if *r* = 20 then the minimum value of the index is 0.9717 and the maximum is 1.0370.

How much difference does chaining make?

The impression gained from Hill (2006) is that GDP growth is much more sensitive to the value of *r* than the results in Table 2 or Figure 4 would suggest. But Hill's results (though using different data) are all based on 2-year, not chained, indices. Figure 5 shows 2-year indices for 1987-2000, 2000-2019 and for the whole period 1987-2019. The 2-year (non-chained) indices use only the weights of the first and last years of their period. Qualitatively the picture seems very similar to Hill's.

Table 5 gives a direct comparison between chained and non-chained (2-year) superlative indices of real GDP for values of *r* ranging from -20 to +20. Three time periods are considered: 1987-2000, 2000-2019, and the whole period 1987-2019. Figure 6 compares 2-year and chained superlative indices directly for the whole period.

The first thing to note is that if *r* lies between about -1 and +1, then the 2-year and the chained indices are very similar, e.g. in the Törnqvist case the estimated growth rates over 1987-2019 are 2.350% p.a. versus 2.402% p.a. But outside that range, i.e. $r < -1$ or $r > 1$, the two types of index start to diverge markedly; e.g. using the 2-year Fisher suggests growth was 2.893% p.a. compared to 2.404% p.a. for the chained Fisher. For $r = 5$ the 2-year index gives a growth rate of 4.788 compared to 2.415% p.a. on the chained measure. And for $r =$ 20, the growth rate over 1987-2019 is 5.361 compared to 2.562% p.a. on the chained measure.

Table 5 also compares a non-chained Laspeyres with a chained Laspeyres and a non-chained Paasche with a chained Paasche. The non-chained Laspeyres, also known as a Lowe index, uses the weights only of the first year of a given period; the non-chained Paasche uses the weights only of the last year of a given period Without chaining, the Laspeyres shows growth at 3.757% p.a. over the whole period, a big difference from the chained measure, 2.481% p.a.

The conclusion is that chaining makes a huge difference to the sensitivity of the estimated growth rates to the choice of *r*. As we have already seen, the chained superlative indices are fairly insensitive to the value of *r*. With chaining, even the Laspeyres index is not be very different from the Fisher: 2.481 versus 2.404% p.a. over 1987-2019. But use of an unchained Laspeyres (or Lowe) index would give us a fundamentally different view of US growth: 3.757% p.a. over 1987-2019 versus 2.404% p.a. according to the chained Fisher. This might be thought sufficient justification in itself for shifting from a fixed base (or Lowe) index to a chained one. Without chaining, the choice between Fisher and Törnqvist also becomes quite consequential: 2.893 versus 2.350% p.a. over 1987-2019.

In summary, the conclusions of Hill (2006) are largely replicated and confirmed for 2-year indices. But equally we see that an opposite conclusion applies to chained indices: they are fairly insensitive to the value of *r*.

5. Conclusions

Adopting the Divisia framework leads naturally to chaining as we have seen and as was clear to Jorgenson and Griliches (1971): see the quote in the introduction. To implement the Divisia approach we need to find good discrete approximations, for which superlative index numbers suggest themselves. But to calculate a superlative index number we have to assume a value for the unknown parameter r in the superlative index number formula. However, using data on real value added from the BEA/BLS industry-level production account, we have found that estimates of real GDP growth are not very sensitive to the value chosen for r, provided that the estimates are chained; if the estimates are not chained, the results can be quite sensitive to the value of r. This is encouraging if we accept chaining since it reduces uncertainty about the true growth rate. We also found that with chaining superlative indices are very close to both value consistency and aggregation consistency (for $-5 \le r \le 5$).

In further work it would be desirable to extend the analysis to the other main aggregates, namely capital, labour and intermediate input. It would also be desirable to extend the time period back before 1987, possibly as far as 1947, if this were to prove possible.

Finally, the estimates presented here rest on the assumption of perfect competition: prices equal marginal costs, though some distortions are still encompassed within the framework, e.g. the price for the same capital or labour input can differ across industries as in Jorgenson et al. (1987). But much of modern macroeconomics is built on the contrary assumption, imperfect competition, at least for short run analysis.[12] Recently there has been much discussion of whether margins are rising; see Basu (2019) for a survey of margin estimates in the United States which vary widely though are generally positive. (Macroeconomists of the real business cycle school still hold to the perfect competition assumption (price equals marginal cost) but they seem to be in the minority.) Extracting estimates of output and productivity from the national accounts is a much more challenging task under imperfect competition since it requires the estimation of margins which are not directly observed (Basu and Fernald 2002). It also raises the possibility that aggregate TFP is affected by movements of resources towards or away from firms with high margins (Baqaee and Farhi 2020).

[12] The literature on imperfect competition and productivity goes back to Hall (1988).

TABLES

Table 1(a)

Shares of 63 industries in U.S. nominal GDP, percent

Number	*Industry*	1987	2000	2019
1	Farms	1.24	0.73	0.55
2	Forestry, fishing, and related activities	0.35	0.21	0.18
3	Oil and gas extraction	0.95	0.65	0.81
4	Mining, except oil and gas	0.38	0.27	0.26
5	Support activities for mining	0.14	0.13	0.24
6	Utilities	2.52	1.72	1.48
7	Construction	4.23	4.40	4.02
8	Wood products	0.40	0.27	0.18
9	Nonmetallic mineral products	0.51	0.41	0.29
10	Primary metals	0.66	0.45	0.29
11	Fabricated metal products	1.39	1.16	0.73
12	Machinery	1.40	1.08	0.75
13	Computer and electronic products	1.98	2.15	1.37
14	Electrical equipment, appliances, and components	0.72	0.44	0.29
15	Motor vehicles, bodies and trailers, and parts	1.43	1.31	0.71
16	Other transportation equipment	1.35	0.68	0.74
17	Furniture and related products	0.36	0.32	0.14
18	Miscellaneous manufacturing	0.49	0.56	0.43
19	Food and beverage and tobacco products	1.82	1.56	1.24
20	Textile mills and textile product mills	0.42	0.27	0.07
21	Apparel and leather and allied products	0.46	0.21	0.04
22	Paper products	0.79	0.59	0.27
23	Printing and related support activities	0.53	0.42	0.18
24	Petroleum and coal products	0.43	0.50	0.73
25	Chemical products	1.84	1.79	1.74
26	Plastics and rubber products	0.66	0.63	0.37
27	Wholesale trade	5.74	5.94	5.68
28	Retail trade	6.94	6.54	5.19
29	Air transportation	0.49	0.55	0.65
30	Rail transportation	0.42	0.22	0.19
31	Water transportation	0.08	0.08	0.06
32	Truck transportation	0.90	0.94	0.77
33	Transit and ground passenger transportation	0.15	0.18	0.24
34	Pipeline transportation	0.13	0.09	0.19
35	Other transportation and support activities	0.68	0.63	0.61
36	Warehousing and storage	0.22	0.25	0.34
37	Publishing industries, except internet (includes software)	0.91	1.11	1.30
38	Motion picture and sound recording industries	0.55	0.52	0.39
39	Broadcasting and telecommunications	2.72	2.64	2.09
40	Data processing, internet publishing, and other information services	0.28	0.23	1.27
41	Federal Reserve banks, credit intermediation, and related activities	2.93	3.07	3.22
42	Securities, commodity contracts, and investments	0.83	1.27	1.47
43	Insurance carriers and related activities	1.64	2.62	2.69
44	Funds, trusts, and other financial vehicles	0.15	0.15	0.11
45	Real estate	10.30	10.44	11.13
46	Rental and leasing services and lessors of intangible assets	1.04	1.30	1.19
47	Legal services	1.20	1.23	1.26
48	Computer systems design and related services	0.43	1.09	1.65
49	Miscellaneous professional, scientific, and technical services	2.86	3.90	4.41

50	Management of companies and enterprises	1.55	1.63	1.83

Table 1(a), continued.

51	Administrative and support services	1.53	2.44	2.74
52	Waste management and remediation services	0.22	0.25	0.26
53	Educational services	0.67	0.91	1.22
54	Ambulatory health care services	2.42	2.75	3.48
55	Hospitals and nursing and residential care	2.18	2.47	2.99
56	Social assistance	0.32	0.50	0.64
57	Performing arts, spectator sports, museums, and related activities	0.32	0.47	0.65
58	Amusements, gambling, and recreation industries	0.35	0.47	0.43
59	Accommodation	0.73	0.89	0.82
60	Food services and drinking places	1.68	1.85	2.17
61	Other services, except government	2.43	2.67	2.02
62	Federal	6.86	4.74	4.78
63	State and local	9.69	10.08	11.74
	TOTAL (GDP)	100.0	100.0	100.0
	Min	0.08	0.08	0.04
	Max	10.30	10.44	11.74
	S.D.	2.11	2.11	2.25

Table 1(b)
Value added shares of 9 industry groups in GDP, percent

Industry group no	Industry group code	Industries	Industry group name	1987	2000	2019
1	AFFHM	1-5	Agriculture, forestry, fishing, hunting, and mining	3.06	1.99	2.04
2	TWU	6, 29-36	Transportation, warehousing, utilities	5.59	4.65	4.54
3	CONST	7	Construction	4.23	4.40	4.02
4	MANUF	8-26	Manufacturing	17.63	14.79	10.55
5	TRADE	27,28	Trade	12.68	12.48	10.87
6	INFO	37-40	Information	4.46	4.50	5.05
7	FIRE	41-46	Finance, insurance, real estate, rental and leasing	16.89	18.84	19.82
8	OSERV	47-61	Other services	18.9	23.53	26.58
9	GOV	62,63	Government	16.55	14.82	16.53
			TOTAL (GDP)	100.00	100.00	100.00

Source U.S. Bureau of Economic Analysis, BEA/BLS Integrated Industry-level Production Account (BEA-BLS-industry-level-production-account-1987-2020.xlsx, released May 11 2022).
Note Industry-level shares are industry value added as % of nominal GDP.

Table 2
Chained superlative indices: annual growth rates of real GDP in the U.S., 1987-2019, % p.a.

Value of *r*	Mean	Std. Dev.	Min	Max
-20	2.547	3.193	-12.009	9.782
-19	2.542	2.981	-11.111	8.947
-18	2.537	2.771	-10.165	8.120
-17	2.531	2.564	-9.180	7.317
-16	2.525	2.365	-8.169	6.559
-15	2.518	2.178	-7.151	5.863
-14	2.510	2.008	-6.149	5.241
-13	2.500	1.861	-5.187	4.699
-12	2.489	1.742	-4.290	4.472
-11	2.477	1.654	-3.476	4.473
-10	2.465	1.594	-2.759	4.513
-9	2.453	1.559	-2.142	4.538
-8	2.442	1.543	-1.695	4.552
-7	2.432	1.539	-2.062	4.560
-6	2.423	1.540	-2.362	4.562
-5	2.416	1.545	-2.599	4.562
-4	2.411	1.550	-2.780	4.559
-3	2.407	1.554	-2.914	4.556
-2	2.404	1.558	-3.008	4.554
-1	2.402	1.560	-3.068	4.552
0	2.402	1.560	-3.099	4.551
1	2.402	1.559	-3.104	4.552
2	2.404	1.557	-3.083	4.554
3	2.406	1.554	-3.036	4.557
4	2.410	1.549	-2.961	4.562
5	2.415	1.542	-2.854	4.567
6	2.421	1.534	-2.710	4.573
7	2.429	1.526	-2.525	4.577
8	2.438	1.517	-2.290	4.580
9	2.448	1.510	-2.002	4.580
10	2.460	1.508	-1.661	4.574
11	2.473	1.515	-1.613	4.560
12	2.486	1.538	-2.138	4.534
13	2.500	1.582	-2.758	4.493
14	2.512	1.651	-3.472	4.502
15	2.524	1.748	-4.270	4.538
16	2.534	1.872	-5.138	4.935
17	2.542	2.019	-6.054	5.515
18	2.549	2.185	-6.996	6.165
19	2.556	2.366	-7.943	6.872
20	2.562	2.556	-8.876	7.621
Memo items				
Limit as $|r| \to \infty$	2.994	10.102.	.-22.421	22.829
Chained Laspeyres	2.481	1.535	-2.692	4.655
Chained Paasche	2.327	1.583	-3.474	4.453

Source U.S. Bureau of Economic Analysis, BEA/BLS Integrated Industry-level Production Account. (BEA-BLS-industry-level-production-account-1987-2020.xlsx, released May 11 2022).
Note Quantities are real value added for 63 industries; weights are shares in aggregate nominal value added (nominal GDP). Growth rates calculated as 100 x mean annual log difference over the period. Superlative indices calculated from equations (12) and (19). Limit as $|r| \to \infty$ calculated from equation (13).

Table 3
Value consistency index (1987=1):
Ratio of value index to product of chained price index and chained quantity index

	Value of *r*									Lasp-
year	-20	-5	-2	-1	0	1	2	5	20	-eyres
1987	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1988	1.0006	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0007	0.9990
1989	1.0025	1.0001	1.0000	1.0000	1.0000	1.0000	1.0000	1.0001	1.0022	0.9982
1990	1.0024	1.0002	1.0000	1.0000	1.0000	1.0000	1.0000	1.0001	1.0018	0.9974
1991	1.0064	1.0005	1.0001	1.0001	1.0000	1.0000	1.0000	1.0002	1.0048	0.9967
1992	1.0088	1.0007	1.0002	1.0001	1.0000	1.0000	1.0000	1.0002	1.0064	0.9959
1993	1.0052	1.0005	1.0001	1.0001	1.0000	1.0000	1.0000	1.0001	1.0035	0.9951
1994	1.0095	1.0007	1.0002	1.0001	1.0000	1.0000	1.0000	1.0002	1.0066	0.9939
1995	0.9980	1.0001	1.0001	1.0001	1.0000	1.0000	1.0000	0.9999	0.9971	0.9917
1996	0.9865	0.9994	0.9999	1.0000	1.0000	1.0000	1.0000	0.9997	0.9883	0.9890
1997	1.0109	0.9995	0.9999	1.0000	1.0000	1.0000	1.0000	0.9997	1.0045	0.9870
1998	1.0197	1.0001	1.0001	1.0000	1.0000	1.0000	1.0000	0.9999	1.0111	0.9849
1999	1.0236	1.0003	1.0001	1.0001	1.0000	1.0000	1.0000	1.0000	1.0143	0.9833
2000	0.9274	0.9969	0.9994	0.9998	1.0000	1.0001	1.0000	0.9986	0.9359	0.9794
2001	0.9078	0.9958	0.9991	0.9997	1.0000	1.0001	1.0000	0.9982	0.9211	0.9780
2002	0.9094	0.9960	0.9992	0.9997	1.0000	1.0001	1.0000	0.9982	0.9220	0.9774
2003	0.8825	0.9946	0.9989	0.9996	1.0000	1.0001	1.0000	0.9977	0.9008	0.9760
2004	0.8806	0.9944	0.9988	0.9995	1.0000	1.0001	1.0000	0.9977	0.8994	0.9751
2005	0.8738	0.9939	0.9987	0.9995	1.0000	1.0001	1.0000	0.9975	0.8941	0.9740
2006	0.8687	0.9936	0.9986	0.9995	1.0000	1.0001	1.0000	0.9974	0.8906	0.9727
2007	0.8678	0.9936	0.9986	0.9995	1.0000	1.0002	1.0000	0.9974	0.8897	0.9715
2008	0.9686	0.9997	1.0002	1.0003	1.0003	1.0002	1.0000	0.9987	0.9652	0.9709
2009	1.0854	1.0056	1.0015	1.0008	1.0003	1.0000	1.0000	1.0013	1.0651	0.9634
2010	0.9972	1.0027	1.0008	1.0005	1.0002	1.0001	1.0000	1.0004	1.0008	0.9611
2011	0.9770	1.0016	1.0005	1.0004	1.0002	1.0001	1.0000	1.0000	0.9848	0.9596
2012	0.9761	1.0015	1.0005	1.0003	1.0002	1.0001	1.0000	1.0000	0.9844	0.9588
2013	0.9751	1.0014	1.0005	1.0003	1.0002	1.0001	1.0000	1.0000	0.9838	0.9584
2014	0.9623	1.0010	1.0004	1.0003	1.0002	1.0001	1.0000	0.9999	0.9752	0.9577
2015	1.2004	1.0154	1.0034	1.0015	1.0004	0.9999	1.0000	1.0047	1.1735	0.9545
2016	1.2319	1.0169	1.0039	1.0018	1.0005	0.9999	1.0000	1.0050	1.1911	0.9542
2017	1.2196	1.0162	1.0037	1.0017	1.0005	0.9999	1.0000	1.0048	1.1826	0.9536
2018	1.2128	1.0158	1.0036	1.0017	1.0005	0.9999	1.0000	1.0047	1.1777	0.9528
2019	1.2238	1.0163	1.0037	1.0017	1.0005	0.9999	1.0000	1.0049	1.1861	0.9520

Source U.S. Bureau of Economic Analysis, BEA/BLS Integrated Industry-level Production Account. (BEA-BLS-industry-level-production-account-1987-2020.xlsx, released May 11 2022).

Table 4
Aggregation consistency index (1987=1):

	Value of *r*								
year	-20	-5	-2	-1	0	1	2	5	20
1987	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1988	1.0003	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0002
1989	1.0005	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0004
1990	1.0018	1.0001	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0014
1991	1.0017	1.0001	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0013
1992	1.0020	1.0001	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0013
1993	1.0019	1.0002	1.0001	1.0000	1.0000	1.0000	1.0000	1.0000	1.0012
1994	1.0010	1.0001	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0002
1995	0.9982	1.0001	1.0001	1.0000	1.0000	1.0000	1.0000	0.9999	0.9973
1996	0.9973	1.0001	1.0001	1.0000	1.0000	1.0000	1.0000	0.9999	0.9965
1997	1.0087	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	1.0025
1998	1.0080	1.0001	1.0001	1.0000	1.0000	1.0000	1.0000	0.9998	1.0018
1999	1.0050	0.9998	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9992
2000	0.9977	0.9998	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9936
2001	0.9892	0.9998	1.0000	1.0001	1.0000	1.0000	1.0000	0.9997	0.9890
2002	0.9866	0.9996	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9868
2003	0.9843	0.9995	1.0000	1.0000	1.0001	1.0000	1.0000	0.9997	0.9858
2004	0.9840	0.9995	1.0000	1.0000	1.0001	1.0001	1.0000	0.9997	0.9856
2005	0.9839	0.9995	1.0000	1.0000	1.0001	1.0001	1.0000	0.9997	0.9853
2006	0.9789	0.9993	0.9999	1.0000	1.0001	1.0001	1.0000	0.9995	0.9816
2007	0.9773	0.9990	0.9999	1.0000	1.0001	1.0001	1.0000	0.9995	0.9802
2008	1.0558	1.0005	1.0001	1.0001	1.0001	1.0001	1.0000	0.9997	1.0370
2009	1.0303	0.9981	0.9997	1.0000	1.0000	0.9999	0.9997	0.9981	1.0127
2010	0.9775	0.9977	0.9996	0.9999	1.0000	1.0000	0.9997	0.9981	0.9764
2011	0.9784	0.9977	0.9996	0.9999	1.0000	1.0000	0.9998	0.9981	0.9774
2012	0.9767	0.9976	0.9996	0.9999	1.0000	1.0000	0.9997	0.9981	0.9759
2013	0.9755	0.9975	0.9996	0.9999	1.0000	1.0000	0.9997	0.9980	0.9750
2014	0.9695	0.9975	0.9996	0.9999	1.0000	1.0000	0.9998	0.9981	0.9718
2015	0.9756	0.9976	0.9996	0.9999	0.9999	0.9998	0.9995	0.9975	0.9717
2016	0.9855	0.9978	0.9997	0.9999	1.0000	0.9998	0.9995	0.9975	0.9773
2017	0.9853	0.9978	0.9997	0.9999	1.0000	0.9998	0.9995	0.9975	0.9774
2018	0.9839	0.9977	0.9997	0.9999	1.0000	0.9998	0.9995	0.9975	0.9764
2019	0.9832	0.9977	0.9997	0.9999	1.0000	0.9998	0.9995	0.9974	0.9756
Std. Dev.	0.0195	0.0011	0.0002	0.0001	0.0000	0.0001	0.0002	0.0010	0.0148
Min	0.9695	0.9975	0.9996	0.9999	0.9999	0.9998	0.9995	0.9974	0.9717
Max	1.0558	1.0005	1.0003	1.0002	1.0001	1.0001	1.0000	1.0000	1.0370

Source U.S. Bureau of Economic Analysis, BEA/BLS Integrated Industry-level Production Account. (BEA-BLS-industry-level-production-account-1987-2020.xlsx, released May 11 2022).

Note The aggregation consistency index is the ratio of the 2-step chained quantity index to the 1-step chained quantity index. First step of the 2-step index is chained quantity indices for each of 9 industry groups. The second step aggregates these to the GDP level. See Table 1(b) for the definition of the 9 industry groups.

Table 5
Superlative indices: average annual growth rates of real GDP, % p.a.
2-year indices compared to chained indices

	1987-2000		2000-2019		1987-2019	
Value of *r*	2-year	Chained	2-year	Chained	2-year	Chained
-20	7.822	3.298	4.855	2.033	4.932	2.547
-19	7.819	3.307	4.870	2.018	4.941	2.542
-18	7.810	3.312	4.883	2.006	4.950	2.537
-17	7.793	3.314	4.894	1.996	4.961	2.531
-16	7.768	3.313	4.902	1.986	4.972	2.525
-15	7.732	3.308	4.904	1.978	4.984	2.518
-14	7.680	3.302	4.896	1.968	4.997	2.510
-13	7.609	3.294	4.873	1.958	5.009	2.500
-12	7.512	3.285	4.829	1.945	5.020	2.489
-11	7.383	3.275	4.757	1.931	5.027	2.477
-10	7.211	3.266	4.647	1.917	5.025	2.465
-9	6.983	3.257	4.489	1.902	5.005	2.453
-8	6.682	3.248	4.271	1.889	4.952	2.442
-7	6.287	3.241	3.979	1.878	4.847	2.432
-6	5.780	3.234	3.605	1.869	4.663	2.423
-5	5.165	3.227	3.162	1.861	4.365	2.416
-4	4.501	3.222	2.705	1.856	3.917	2.411
-3	3.915	3.218	2.322	1.852	3.329	2.407
-2	3.509	3.215	2.068	1.849	2.769	2.404
-1	3.291	3.213	1.929	1.848	2.443	2.402
0	3.220	3.212	1.873	1.847	2.350	2.402
1	3.277	3.213	1.876	1.848	2.467	2.402
2	3.484	3.214	1.929	1.849	2.893	2.404
3	3.889	3.216	2.041	1.852	3.612	2.406
4	4.494	3.220	2.223	1.856	4.298	2.410
5	5.192	3.225	2.482	1.861	4.788	2.415
6	5.843	3.230	2.805	1.868	5.115	2.421
7	6.378	3.237	3.152	1.876	5.333	2.429
8	6.793	3.245	3.485	1.885	5.477	2.438
9	7.108	3.254	3.780	1.897	5.567	2.448
10	7.344	3.263	4.029	1.910	5.616	2.460
11	7.520	3.274	4.233	1.925	5.633	2.473
12	7.650	3.285	4.396	1.940	5.625	2.486
13	7.743	3.296	4.524	1.955	5.601	2.500
14	7.808	3.307	4.620	1.969	5.568	2.512
15	7.852	3.317	4.689	1.981	5.532	2.524
16	7.880	3.327	4.738	1.991	5.494	2.534
17	7.895	3.334	4.769	2.000	5.457	2.542
18	7.900	3.340	4.786	2.008	5.423	2.549
19	7.899	3.342	4.794	2.018	5.390	2.556
20	7.892	3.342	4.796	2.028	5.361	2.562
Memo items						
Laspeyres	4.086	3.294	2.217	1.924	3.757	2.481
Paasche	2.881	3.134	1.641	1.775	2.029	2.327

Source U.S. Bureau of Economic Analysis, BEA/BLS Integrated Industry-level Production Account. (BEA-BLS-industry-level-production-account-1987-2020.xlsx, released May 11 2022).
Note Quantities are real value added for 63 industries; weights are shares in aggregate nominal value added (nominal GDP). Growth rates calculated as 100 x mean annual log difference over the stated period. Superlative indices calculated from equations (12) and (19). 2-year superlative indices use weights of just the first and last years of the period; chained superlative indices use weights of all years of the period. 2-year Laspeyres (Paasche) uses only weights of first (last) year of period.

FIGURES

Figure1

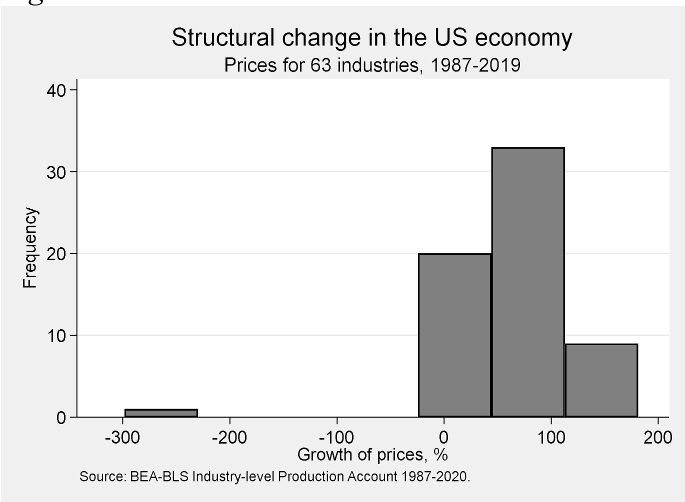


Figure 2

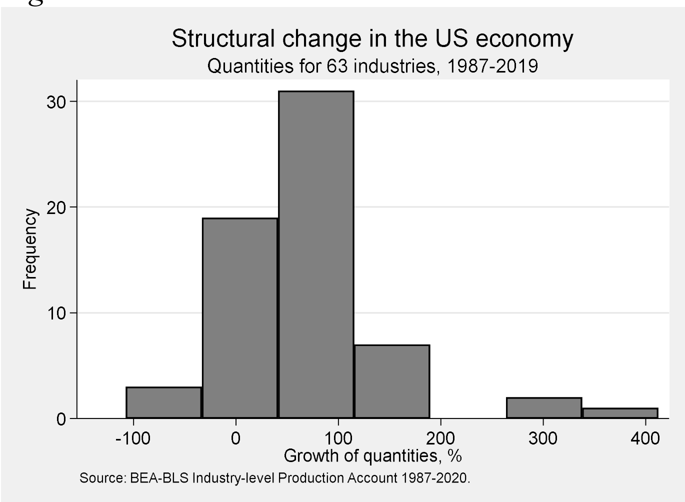


Figure 3

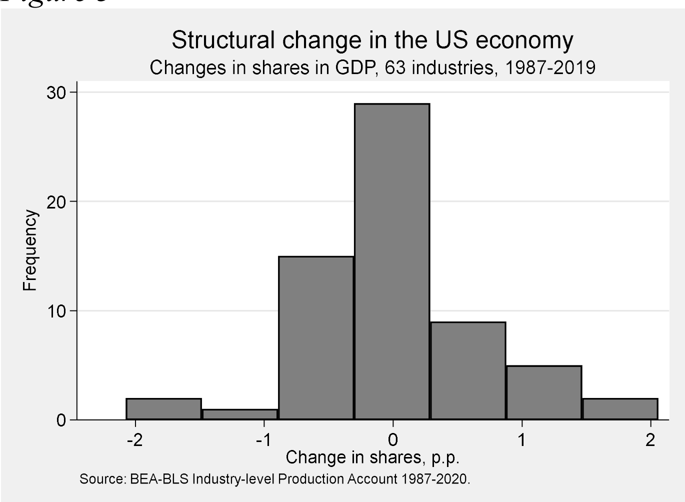
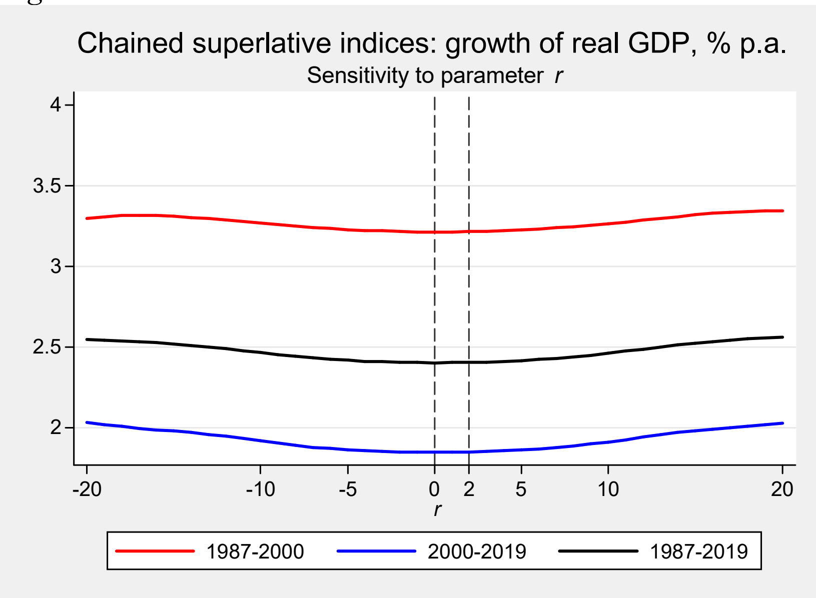
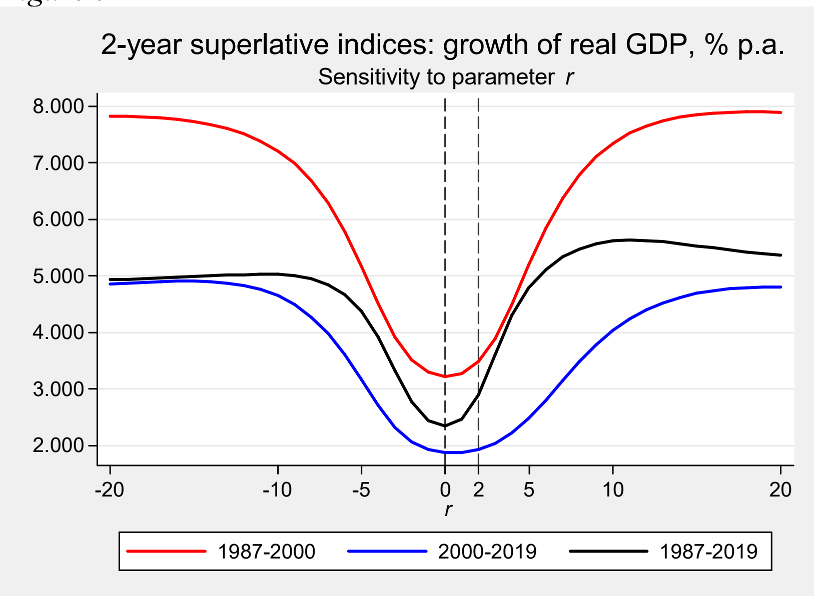


Figure 4



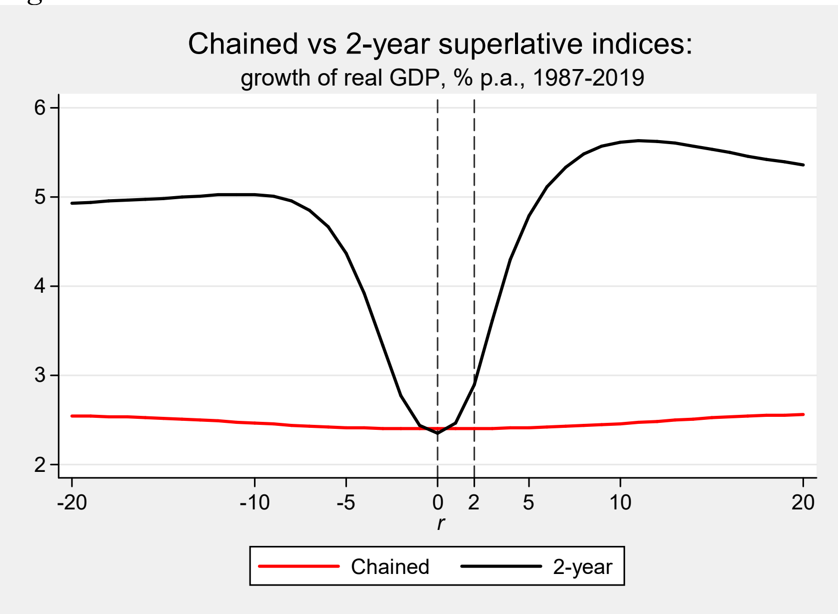
Source Table 2.

Figure 5



Source Table 4.

Figure 6



Source Tables 2 and 4.

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