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September 2022

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Abstract - …..

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1. Introduction

The objective of this paper is to determine and compare the macroeconomic effects of increasing wage subsidies for researchers in Belgium for each industry or group of industries separately, taking into account not only the direct effect of an increased stock of knowledge (or technology) on the sector's own productivity but also the indirect effect such an increase may have on other industries' R&D stocks via knowledge spillovers. To this end, a medium-size dynamic general equilibrium model is developed that allows for a realistic calibration to the Belgian economy and that adopts a semi-endogenous growth framework where R&D investments accumulate into industry-specific knowledge stocks, which can affect each other through an innovation network.

Economic research has generated a lot of evidence that R&D is an important factor in economic growth. Perhaps the most recent comprehensive review of the literature is that of Hall, Mairesse & Mohnen (2010). As the most important channels through which R&D leads to higher productivity and welfare, they identify quality improvement, cost reduction, the increase of the variety of goods and spillover effects between industries and countries. Regarding the latter, they differentiate, following Griliches (1979, 1992), between "pecuniary" or rent spillovers – that follow from a firm or sector's inability to appropriate the entire benefits of their increased R&D, e.g., because of a lack of market power or imitation by other firms – and "non-pecuniary" or knowledge spillovers – emanating from the nonrival and often only partially excludable nature of knowledge. The latter type will be the focus of this paper.

Notwithstanding the fact that estimation of the returns to R&D is fraught with measurement and econometric difficulties, the study suggests a consensus in the literature that the direct, private returns to R&D are strongly positive, regardless of whether a production function or dual cost function approach is used. Output elasticities of R&D in the study range between 0.01 and 0.25 and are centred around 0.08, while rates of return are likely to lie between 20 and 30% in the primal approach and between 10 and 20% in the dual approach.

Regarding spillovers or "social returns", the authors conclude that estimates are generally large, although estimations often seem to suffer from low precision and often operationalize spillovers differently, hampering comparability between studies. Nevertheless, elasticities with respect to external R&D are similar, lying generally between 0.05 and 0.09. The authors report that the most common technique of constructing an aggregate external knowledge stock consists in calculating a weighted arithmetic average of underlying stocks, where the weights can be based on intermediate input transactions, trade intensity between countries, flows of patents, patent citations, technical proximity. Whereas some authors have chosen to estimate effects of different stocks of external knowledge separately, this vectorization approach is prone to leading to multicollinearity issues.

An early example of constructing aggregate stocks of knowledge appears in Coe & Helpman (1995), who want to discriminate between effects of domestic and foreign R&D on productivity. To that end, they define domestic knowledge stocks per country as accumulated real deflated R&D expenditures (assuming a depreciation rate of 0.05) and then construct an aggregate foreign stock per country taking the import weighted arithmetic average of its trade partners' domestic stocks. They find a domestic TFP elasticity of R&D between 0.07 and 0.09 for OECD members that are not in the G7, which increases by another 0.14 to 0.2 points for G7 countries. For foreign knowledge, they estimate the interaction effect of the foreign knowledge stock with the aggregate import share, leading to a significantly positive coefficient of 0.27. Hence, for small, open countries, foreign R&D seems to be at least as important as domestic R&D for increasing TFP. Interestingly, including the log of the foreign knowledge stock to the specification, this "direct effect" of foreign R&D is negative but is compensated by the interaction with import share whenever the latter is higher than 20% (more specifically, the direct effect is -0.11 while the interaction effect is 0.53). This is suggestive of a direct "market stealing" effect, which is compensated by positive spillovers via the trade channel. Given its high import share, Belgium was the country with the highest estimated impact from foreign R&D, out of the group of 22 countries considered in the study.

Bottazzi & Peri (2007) also investigate the effect of foreign knowledge, now on domestic R&D production rather than TFP, acknowledging that TFP, as a rest factor, may catch all sorts of processes (e.g., factor utilization dynamics) besides technology, even when prices and input quantities are measured perfectly. They specify R&D production, measured as the number of new patents, as loglinearly depending on both the existing stocks of foreign and domestic knowledge, as well as on contemporaneous efforts directed to R&D, as measured by the number of researchers employed. In a first step, they rewrite this specification in terms of the instantaneous technology growth rate and establish that the growth rate time series is stationary, providing support for semi-endogenous growth. Combined with the fact that the time series for R&D labour and both knowledge stocks are non-stationary, there must be a long-term cointegration relationship between them, which they estimate in a second step with the domestic R&D stock as a dependent variable. This results in significant estimates of the elasticity of the domestic knowledge stock with respect to foreign knowledge between 0.17 and 0.56 depending on the exact specification. Knowledge stocks were constructed using a perpetual inventory method (PIM) with depreciation rate 0.1 applied to the number of patents rather than R&D expenditures, inter alia avoiding timing issues between R&D investments and outcomes[2]. Aggregate foreign stocks are calculated as simple averages, not weighted by import shares. The results of the study were replicated by Bottasso et al. (2015), who also showed that they are robust to a different specification taking into account cross-sectional correlation, although they see a somewhat more important role for international spillovers.

Several authors have refined these results by differentiating between industries. Verspagen (1997) estimated domestic intra- and interindustry as well as foreign spillovers on labour productivity using sectoral data on a sample of 13 countries, also distinguishing between knowledge and rent spillovers via the weights used in constructing aggregate knowledge stocks (citations in patents vs. intermediate consumption shares) and finds evidence for both types. His estimate of domestic intra-industry knowledge spillovers is 0.10, with lower values for interindustry (0.03) and foreign (0.05) spillovers. The estimates for rent spillovers are similar, with higher values for interindustry (0.06) and foreign (0.08) spillovers. High-tech industries seem to have larger intra-industry effect, while low-tech industries seem to receive relatively larger foreign than domestic spillovers.

[2] Their other argument, that patents are more adequate than R&D expenditures to capture the external effects of R&D since R&D resources like scientists and labs are excludable, seems somewhat lopsided since the raison d'être of patents is of course to make the outcomes of R&D investment more excludable.

Keller (2002) studied the effect of the trade in intermediate goods on the transmission of R&D spillovers on the TFP of other industries and countries, within the framework of an expanding variety model on the level of intermediates. For 13 manufacturing industries in 8 OECD countries (Canada, France, Germany, Italy, Japan, Sweden, UK, US) in the period between 1970 and 1991, he constructed cumulative stocks of R&D expenditures using a PIM with a depreciation rate of 0.1. As explanatory variables, 4 aggregate industry-specific stocks of knowledge were defined as potential sources of R&D spillovers: domestic and foreign stocks of both intra- and interindustry knowledge. These aggregates again were calculated as weighted average of underlying stocks, where the weights were based on both import shares and intermediate consumption shares (of the US, applied to all countries). Complementary to the trade-based approach, Keller also weighted stocks using a "technology flow matrix", which contains in each cell the proportion of R&D used in the row industry that originated from the column industry (so that row sums sum to 1). The results indicate that technology transmission through input-output and import relations is important. Domestic intra-industry knowledge has a TFP elasticity of 0.21, comparable in size to the effect of foreign intra-industry knowledge, whereas the effect of domestic inter-industry knowledge is more than twice this size. There seems to be no significant foreign interindustry effect. The apparently strange phenomenon of interindustry effects dominating intra-industry spillovers leads Keller to conclude that the pure "transactions" view of intersectoral technology diffusion does not capture the entire diffusion process. The technology flows method leads to a domestic intra-industry elasticity of 0.10, with the interindustry effect 25% higher and the foreign intra-industry effect 15% lower. Strangely, the foreign interindustry elasticity is negative, almost twice the size, in absolute terms, of the domestic intra-industry effect. In a recent review, Keller (2021) concludes that despite econometric issues in identifying spillovers, there is by now robust evidence that both trade and foreign direct investment result in substantial knowledge spillovers.

In a similar exercise, Frantzen (2002) estimated domestic and foreign spillovers on TFP for 14 OECD countries from 1972 to 1994, finding that both types of spillovers matter, though the effect of domestic spillovers is slightly higher than that of foreign spillovers (0.34 vs. 0.23 respectively). He then further differentiated between intra- and interindustry knowledge, leading to the conclusion that the latter dominates the former, the difference being more pronounced for foreign spillovers (0.14 and 0.24 vs. 0.05 and 0.18 respectively).

As for the existence of spillovers in Belgium, Biatour, Dumont & Kegels (2011) estimated a model to find the main determinants of TFP for 21 industries in the period 1998-2007. As in other studies, they constructed industry knowledge stocks by accumulating R&D expenditures in constant prices with a depreciation rate of 0.15. These were aggregated into 6 industry-specific aggregates: 4 trade-related aggregates as in Keller (2002), with intermediate consumption and import shares as weights, and a domestic and foreign aggregate based on technological proximity weights, i.e., international patent citations as in Verspagen (1997). Overall, they find a significantly positive effect for positive domestic interindustry rent spillovers (via the trade channel) and for foreign R&D knowledge spillovers (via the patent citation channel), amounting to a TFP elasticity of 0.08 and 0.11 respectively. Distinguishing between manufacturing and services, the former group also displays significantly positive spillovers for domestic intra-industry (with an elasticity of 0.14). Furthermore, they performed estimations separately for low-, medium- and high-tech manufacturing, dropping the foreign knowledge aggregate based on patent citations in response to collinearity issues. They found that the domestic intra-industry effect is limited to the high-tech manufacturing industries, while positive domestic interindustry spillovers and negative domestic knowledge spillovers are at work in the medium- and high-tech industries. Positive foreign interindustry spillovers are observed for medium-tech manufacturing only, whereas low-tech manufacturing experiences negative spillovers from foreign intra-industry R&D, possibly pointing to market stealing effects.

All these studies point to the existence of significant, mostly positive spillovers from R&D. However, a recent meta-analysis by Ugur, Churchill & Luong (2020) suggests that the effect may be much smaller when accounting for selection bias, low statistical power and observed sources of heterogeneity between studies, like the inclusion of controls for own R&D capital and for whether data are collected on the firm, industry or country level (estimates being lower on the industry level, possibly due to greater creative destruction and market stealing effects). The authors conclude that the effect of spillovers is smaller than previous research suggested and smaller than the effects of own R&D (with estimates of the former effects less precise than those of the latter), but that they are still significantly positive, especially for OECD countries that have a longer history of investment in own R&D, suggesting that investing in own R&D also increases the absorptive capacity for external knowledge[3]. They estimate the elasticity of productivity with respect to knowledge spillovers to be 0.07, but only 0.01 when only the adequately powered studies are taken into account.

In a recent review of the literature on domestic and international R&D spillovers, Belderbos & Mohnen (2020) point at measurement issues and the importance of the choice of potential spillover channels, which may explain some of the diverging conclusions among empirical studies. They argue that spillover matrices should be sufficiently broad to capture their correlated effects and suggest using matrices based on patent citations, between sectors of use (citing patents) and sectors of origin (cited patents). These allow to include knowledge spillovers from manufacturing industries to services, which are generally difficult to incorporate but important in the context of macro analysis.

The presence of positive domestic spillovers may justify government intervention to internalize the external effect, provided the government disposes of adequate instruments and the marginal cost of public funds is not too high[4]. But for policy purposes, especially regarding the allocation of means over different industries, the studies described above may not contain enough detail. Indeed, at best they arrive at separate estimates for 4 categories of spillovers: domestic intra-industry, domestic interindustry, foreign intra-industry and foreign interindustry. Not only are these categories highly aggregated, but they are also often the same for all industries, so that industries are more or less symmetric in that they have the same own-industry and cross-industry R&D effects and, hence, are equally important to the generation of knowledge. The situation where one industry occupies a more central role in knowledge production and has relatively more influence on other industries, is precluded by this structure. As mentioned above, there are good reasons for this lack of detail and asymmetry, in the first place the multicollinearity issues that show up when including too much different knowledge stocks as explanatory variables.

[3] Cohen & Levintahl (1990) already pointed out that own R&D activities and investment in human capital are not only important to create knowledge but also to enhance the capacity to absorb the knowledge created in external R&D activities.
[4] Although some degree of internalization may arise spontaneously through "multisector innovation hubs", see Liu & Ma (2022).

To address the issues of centrality of an industry and optimal resource allocation, an "innovation network" approach may be useful. Acemoglu et al. (2016) define an innovation network as the matrix containing patent citations patterns in the same way as Verspagen (1997), i.e., rows contain citing technology classes and columns contain cited technology classes, while each cell contain the proportion of total patent citations by the row class that refer to patents of the column class (so that all rows sum to 1). Constructing this network on the basis of citation data of US patents during the period 1975-1994 for 36 technology classes, they interpret these proportions as the effect of a 1 patent increase of the cited class on the citing class[5] and use this to project patent formation in the next 10 years (1995-2004). It turns out that patent growth in upstream industries has strong predictive power on downstream innovation: it explains 55% of variation in patenting levels. After correcting (1) for persistence in relative sizes of technological fields and (2) for aggregate fluctuations in annual patenting rates of all fields by panel regression including fields and time controls, this strong relationship between predicted and actual patenting levels remains. Other useful observations in this paper are that the innovation network is stable across time and that many high-profile technology areas, like pharmaceutical products, are at the periphery of the network, i.e., they are knowledge-intensive, but the knowledge they produce does not spread far beyond the own technology class.

Liu & Ma (2022) take the idea of an innovation network further and try to figure out how innovation resources should be allocated across industries optimally, from the perspective of either consumer welfare (operationalized by an intertemporal utility function including consumption and time preference) or the maximization of the long-term growth rate. As in Acemoglu et al. (2016), they define an innovation network based on patent citation shares, but they add an international dimension, using information on Google Patents from more than 40 patent bureaus worldwide, encompassing 36 million patents over the period 1976-2020. The dominant eigenvector in this network is 1 and the associated left eigenvector normalized so that it sums to 1 is called the innovation centrality of the network. This is an important notion in their analysis, as it represents the "*extent to which an industry's R&D activities contribute to economic growth, taking network effects into account*" (Liu & Ma, 2022).

Their model is a multi-industry quality-ladder growth model that includes a production function of new R&D where the output elasticity of R&D workers is equal to 1 and the sum of spillover coefficients per industry is 1. Hence, growth does not obey restrictions like the one in Jones (1995a) or Bottazzo & Peri (2007), so it is endogenous rather than semi-endogenous (though the authors mention that many of their derivations still hold under semi-endogenous growth). Consumers have an intertemporal utility function dependent on aggregate consumption in each period and a discount parameter, where each contemporaneous utility function is a Cobb-Douglas aggregate of the products from different industries. Each industry good is itself a composite of a continuum of varieties of intermediate products, where the latter are produced using a production function linear in the number of workers. R&D increases the quality of the different varieties composing the intermediate goods. They assume that there is a fixed stock of final production workers and of R&D workers, which can both be allocated over the different industries.

[5] Note that they further refine the network to take account for time lags up to 10 years.

Under their assumptions, the optimal R&D allocation under consumer welfare maximization is given by weighted average of the innovation centrality vector and the consumer expenditure shares (which are constant under the Cobb-Douglas utility function), indicating a trade-off between investment and consumption that is modulated by the time preference of consumers, the intensity of the R&D process and the importance of spillovers between industries and countries. The optimal allocation can be reached by a benevolent planner or, in a decentralized economy, using industry-specific R&D taxes with an associated lump-sum tax on consumers. In the absence of corrective taxation, the decentralized equilibrium in their model allocates both R&D and final production workers to industries according to consumer expenditure shares. When the objective is to maximize the long-term growth rate (i.e., when the planner is infinitely patient), the optimal allocation is equal to the innovation centrality of the innovation network.

The goal of this paper, as stated in the beginning of this section, is then to implement the innovation network for Belgium obtained by Liu & Ma (2022) and combine this with a realistic calibration of the Belgian economy, including industry-specific labour markets and capital markets, and a semi-endogenous growth framework to answer the question which industry should receive additional R&D support to increase long-term GDP the most and what are the macroeconomic effects of such a measure.

The structure of this paper is as follows. Section 2 describes DynEMItE, the model used for the simulations, while section 3 discusses the underlying data sources and calibration of the model. The simulation itself and its results are presented in section 4. Section 5 concludes.

2. Model description

The model used for this exercise, DynEMItE, is a dynamic general equilibrium model with multiple industries and semi-endogenous growth. Many of the model specifications are inspired on the Belgian version of QUEST III R&D, a DSGE model developed by the European Commission's Directorate-General for Economic and Financial Affairs (see, e.g., the appendix of D'Auria et al., 2009, for a technical description of the model). Heterogeneity was added to the production side, as the economy exists of n industries with a complete input-output network instead of 1 aggregate sector. The number of industries n can be chosen in function of the exercise and data availability.

As in QUEST III R&D, growth is supposed to be semi-endogenous, meaning that the long-term growth rate is exogenous while the short-term growth rate may be influenced by R&D efforts. This follows empirical evidence by Jones (1995b) and Bottazzo & Peri (2007). The most compelling reason to opt for a semi-endogenous rather than an endogenous growth model is, as, e.g., Jones (1995a) points out, that the latter predict the existence of strong scale effects: increasing the number of researchers by a factor should increase the long-term growth rate by the same factor. This is clearly contradicted by the facts in that the number of researchers in the US and Europe during the last decades has known a large increase, whereas the per capita growth rate has remained constant at best. Jones (1995a) develops a semi-endogenous model characterized by weak scale effects: productivity is affected by efforts devoted to R&D, in that hiring more researchers increases the level of the R&D stock, and hence its short-term growth, but the long-term growth rate remains constant. DynEMItE uses this framework, but introduces different, industry-specific R&D stocks, connected to each other through spillovers in an innovation network. The long-term growth rate is assumed to be the same in every industry, guaranteeing balanced growth, but a differential treatment of R&D across industries may nevertheless change the relative size of the R&D stocks, and indeed of production, between industries.

There are six building blocks in the model: the household sector, production consisting of n different industries, research and development consisting of n industry-specific R&D sectors, government, the (dis)aggregation tree of demand categories and the block of constraints that contains international exchanges between the model's 3 regions – Belgium (BE), the rest of the Eurozone (EA) and the rest of the world (RW) – and the resource constraints derived in a way consistent with the income constraints of the different blocks. Dynamic general equilibrium models are structural models in the sense that they are derived from behavioural rules characterising the different actors. In the first 3 blocks, this takes the form of a Lagrangian function representing the agents' objectives, from which the behavioural rules are derived as first-order conditions (FOCs) in the assumption that agents optimise their behaviour, rationally and intertemporally, with respect to their objectives. The latter 3 blocks do not contain optimizing agents but are described by accounting rules. The remainder of this section contains a description of the main characteristics of each block. The appendix in section 7 provides a more technical description.

In the current version of the model, there is only one representative household. It maximizes intertemporal utility over consumption and labour supply. The former is subject to habit formation, while the latter is split over the n different industries and 3 skill levels per industry, implying that the household's time is divided into $3n$ exogenous shares that reflect the shares of the active population in the respective industries. Hence, there is a separate labour market per industry and skill level, with a specific labour supply decision and a specific wage, reflecting the reality of inter-industry wage differentials (see, e.g., Plasman et al., 2006, for evidence in the Belgian context, and Borjas & Ramey, 2000, who claim that the main market response to an industry wage differential often does not consist in a reallocation of labour but in capital deepening in the high-wage industries). As a consequence, shifts of workers between industries or skill levels can only be introduced in the model exogenously. The 3 skill levels (high, medium and low) are based on educational attainment data in the ISCED classification scheme, which gives of course a rather coarse description of the distribution of skills in society, ignoring the role of experience and upskilling.

The household maximizes its utility under an income constraint: it earns income from labour, capital (both R&D and other), interest on government debt and social benefits, which is spent on consumption, taxes, purchase of government bonds and investment in different (industry-specific) types of capital (again, both R&D and other types of capital). As the shareholder of the firms, the household also receives any economic profits that arise from imperfect competition.

The production sector is composed of n different industries, each of which is assumed to consist of a continuum of individual firms indexed on the unit interval. The individual firms' production functions take a Cobb Douglas form and are assumed to display constant returns to scale (CRS) for labour, capital and intermediate inputs. Public capital plays a productive role, with an industry-specific output elasticity, while the industry-specific stock of R&D capital enters in a labour-augmenting way, in line with the steady state growth theorem (cf. Uzawa, 1961 and Jones & Scrimgeour, 2005). The industries' aggregate production functions are conceived as constant-elasticity-of-substitution (CES) aggregates of individual firms' products, indicative of a monopolistically competitive market structure, where the elasticities – and respective degree of market power – can differ between industries. The aggregate production function inherits CRS with respect to the 3 inputs.

Each industry's labour demand is specified as a CES aggregate of the demand for the 3 skill levels. Most empirical estimates of the associated elasticity of substitution exceed 1 but not by much (often around 2), so that skill levels can be considered imperfect substitutes in production.

The R&D production function is based upon the aforementioned empirical model by Bottazzi & Peri (2007), with some important modifications. Their specification treats (aggregate) innovation in a country as the outcome of a process using both R&D labour and already existing domestic and foreign stocks of R&D capital. Investment in R&D in country r at time t, $ird_{r,t}$, can then be written as:

$$ird_{r,t} = \nu_r \, A_{r,t-1}^{\phi} \, A_{-r,t-1}^{\xi} \, LRD_{r,t}^{\lambda_{rd}}$$

where t denotes the time period, ν_r is a country-specific "efficiency" parameter, $A_{r,t-1}$ is the existing stock of R&D capital in country r, $A_{-r,t-1}$ is the existing stock of R&D capital in the rest of the world (excluding country r) and $LRD_{r,t}$ is the number of researchers in r. ϕ, ξ and λ_{rd} are the relative R&D output elasticities of these 3 variables. Note that ξ can be interpreted as a spillover coefficient since it captures the effect on foreign R&D on country r's investment potential. The inclusion of existing stocks of knowledge implies that the specification captures a "standing upon the shoulders of giants" effect. As described in the introduction, Bottazzi & Peri (2007) construct knowledge stocks as the accumulated number of patents in the respective countries. This method seems able to capture both process and product innovation.

The foreign stock $A_{-r,t-1}$ is calculated as a sum over all other countries' stocks of R&D. As mentioned in the introduction, often in the literature aggregate stocks of knowledge are composed by taking weighted arithmetic means of the component stocks (either per country or per industry or a combination of both). This is somewhat awkward, since ideally one would want to know the elasticity with respect to each component stock separately. The use of aggregate stocks in estimation can often be reduced to multicollinearity issues. But specifying aggregate stocks in terms of geometric weighted means would be more consistent with the underlying intention of identifying effects per component. To clarify this, suppose $A_{-r,t-1}$ in the specification above is defined as a weighted geometric mean of individual countries' stocks: $A_{-r,t-1} = \prod_{k \neq r} A_{k,t-1}^{w_{r,k}}$with $\sum_{k \neq r} w_{r,k} = 1$. If estimations yield an aggregate elasticity of $\hat{\xi}$, the elasticity of an underlying stock $A_{k,t-1}$ can be consistently identified with $\hat{\xi} w_{r,k}$. However, with a weighted arithmetic mean $A_{-r,t-1} = \sum_{k \neq r} w_{r,k}\, A_{k,t-1}$, R&D production will be a function of $\left(\sum_{k \neq r} w_{r,k}\, A_{k,t-1}\right)^{\hat{\xi}}$, from which the output elasticity of component stocks cannot be disentangled.

For this reason, and because of the importance for our exercise of identifying potentially asymmetrical effects of industry-level knowledge stocks due to the consequences of network centrality on optimal taxation, we have slightly modified our specification:

$$ird_{ri,t} = \nu_{ri} \prod_{s=1}^{R} \prod_{j=1}^{n} A_{sj,t-1}^{\varphi_{ri,sj}} \;\; LRD_{ri,t}^{\lambda_{rd,ri}}$$

where the product now runs over all combination of countries $s \in \{1, \dots, R\}$ and industries $i \in \{1, \dots, n\}$. This specification is consistent with, e.g., the specification of Liu & Ma (2022), which allows us to calibrate our spillover coefficients using their observed innovation network. Note that elasticities $\varphi_{ri,sj}$ are allowed to vary according to the "receiving" industry. A necessary condition for balanced, semi-endogenous growth with long-term growth rate γ_A is[6]:

$$\sum\nolimits_{s=1}^{R} \sum_{j=1}^{n} \varphi_{ri,sj} = 1 - \lambda_{rd,ri} \frac{\log(1 + npop)}{\log(1 + \gamma_A)}$$

This condition is used to reweigh estimates for the spillovers $\varphi_{ri,sj}$ during the calibration of the model.

The research sector optimizes its profits:

$$PRD_{ri,t}\, ird_{ri,t} - (1 - subs_{ri})\, W_{h,ri,t}\, LRD_{ri,t}$$

[6] This can be derived as follows. Dividing the accumulation equation $A_{ri,t} = (1 - \delta_A)\, A_{ri,t-1} + ird_{ri,t}$ by $A_{ri,t-1}$ gives $\gamma_{A_{ri},t} + \delta_A = \nu_{ri} \prod_{s=1}^{R} \prod_{j=1}^{n} \left(\frac{A_{sj,t-1}}{A_{ri,t-1}}\right)^{\varphi_{ri,sj}} A_{ri,t-1}^{\sum_{s,j} \varphi_{ri,sj} - 1} LRD_{ri,t}^{\lambda_{rd,ri}}$, with instantaneous industry-specific growth rate $\gamma_{A_{ri},t} = \frac{A_{ri,t}}{A_{ri,t-1}} - 1$. On a balanced growth path, the left-hand side and the relative knowledge stocks within the product sign on the right-hand side are constants so that $A_{ri,t-1}^{\sum_{s,j} \varphi_{ri,sj} - 1} LRD_{ri,t}^{\lambda_{rd,ri}}$ must be a non-zero constant. Since $A_{ri,t}$ evolves as $\tilde{A}_{ri}\, (1 + \gamma_A)^t$ and $LRD_{ri,t}$ as $\widetilde{LRD}_{ri}\, (1 + npop)^t$ for stationary values $\tilde{A}_{ri}$ and $\widetilde{LRD}_{ri}$ on a balanced growth path, it follows that $\left((1 + \gamma_A)^{\sum_{s,j} \varphi_{ri,sj} - 1} (1 + npop)^{\lambda_{rd,ri}}\right)^t$ must also be a non-zero constant, which is only possible for $(1 + \gamma_A)^{\sum_{s,j} \varphi_{ri,sj} - 1} (1 + npop)^{\lambda_{rd,ri}} = 1$ or $1 - \sum_{s,j} \varphi_{ri,sj} = \lambda_{rd,ri} \frac{\log(1+npop)}{\log(1+\gamma_A)}$.

where $PRD_{i,t}$ denotes the price of the R&D investment good and $W_{h,ri,t}$ the wage of the high-skilled workers in industry ri, while $subs_{ri}$ represents an implicit wage subsidy rate for researchers. Note that there is a separate labour market for researchers per industry, as was the case for production. However, in each industry, there is competition between production and R&D over the high-skilled workers, which is apparent from the equation above because the wage of researchers equals that of the high-skilled in production. Since existing stocks of capital act as externalities, production can only be optimized through the number of researchers, which generates the first-order condition (FOC):

$$(1 - subs_{ri})\, W_{h,ri,t} = PRD_{ri,t}\, \lambda_{rd,ri}\, \nu_{ri} \prod_{s=1}^{R} \prod_{j=1}^{n} A_{sj,t-1}^{\varphi_{ri,sj}}\, LRD_{ri,t}^{\lambda_{rd,ri}-1}$$

The inclusion of R&D capital in the firms' production function in a labour-augmenting way according to the steady state growth theorem has 2 important repercussions. In a simplified form, dropping time, industry and country subscripts, the production function can be expressed as:

$$Y = K^{(1-\sigma^M)(1-\alpha)} \left(\frac{A}{1+\gamma_A} L\right)^{(1-\sigma^M)\,\alpha} M^{\sigma^M}$$

with Y representing production, K non-R&D capital, A R&D capital, L the labour composite and M intermediate consumption. Constants σ^M and α are defined so that σ^M denotes the output elasticity of intermediate consumption and $(1-\sigma^M)\,\alpha$ the output elasticity of labour, and production is CRS in the 3 "classic" inputs. γ_A is the long-term technological growth rate. The FOCs with respect to these inputs are:

$$W\,L = (1-\sigma^M)\,\alpha\, MC\, Y \tag{1}$$

$$r\,PI\,K = (1-\sigma^M)\,(1-\alpha)\, MC\, Y \tag{2}$$

$$P^M\, M = \sigma^M\, MC\, Y \tag{3}$$

where W denotes the aggregate wage, r the rental rate of non-R&D capital, PI the deflator of non-R&D investment, P^M the deflator of intermediate consumption and MC the marginal cost, which, under the assumption of CES, is a constant fraction of the price P in steady state, determined by the elasticity of substitution:

$$\frac{P}{MC} = \frac{\sigma}{\sigma - 1} \tag{4}$$

It follows readily from the 3 FOCs above that the total cost of the 3 classic inputs is:

$$W\,L + r\,PI\,K + P^M M = MC\,Y \tag{5}$$

But the firm must also pay for the rental of R&D capital, produced by the research sector, so that total costs amount to:

$$TC = W\,L + r\,PI\,K + P^M M + r_A\,PRD \frac{A}{1+\gamma_A} = MC\,Y + r_A\,PRD \frac{A}{1+\gamma_A} \quad \textbf{(6)}$$

with r_A the rental rate of R&D capital and PRD the deflator of R&D investment, as before. The FOC associated with R&D capital is:

$$r_A\,PRD \frac{A}{1+\gamma_A} = (1-\sigma^M)\,\alpha\,MC\,Y \quad \textbf{(7)}$$

Assuming that in the long run, total income must be higher than total costs, leads to the sustainability criterion that:

$$P\,Y = \frac{\sigma}{\sigma-1} MC\,Y \geq TC$$

Combining this with **(6)** and **(7)** yields:

$$\frac{\sigma}{\sigma-1} MC\,Y \geq MC\,Y + r_A\,PRD \frac{A}{1+\gamma_A} = (1+(1-\sigma^M)\,\alpha)\,MC\,Y$$

In other words, economic sustainability leads to a constraint on the model's parameters:

$$\frac{\sigma}{\sigma-1} \geq 1+(1-\sigma^M)\,\alpha$$

A second issue can be best described by combining the FOCs **(1)** and **(7)**:

$$W\,L = r_A\,PRD \frac{A}{1+\gamma_A} \quad \textbf{(8)}$$

In steady state, the stock A can be substituted by R&D investment IRD using the accumulation equation:

$$A = \frac{1-\delta_A}{1+\gamma_A} A + IRD \;\Rightarrow\; \frac{IRD}{\gamma_A+\delta_A} = \frac{A}{1+\gamma_A}$$

with δ_A the depreciation rate of R&D capital. Substituting this into **(8)** and dividing by total output yields:

$$\frac{WL}{PY} = \frac{r_A}{\gamma_A+\delta_A} \frac{PRD\,IRD}{PY}$$

This relation states that the R&D intensity $\frac{PRD\,IRD}{PY}$ in an industry is a fixed proportion of the labour income share $\frac{WL}{PY}$. To give an intuition, for reasonable values $r_A = 0.07$, $\gamma_A = 0.00375$ and $\delta_A = 0.05$, the R&D intensity would have to be in the order of 75% of the labour income share. Clearly, this is much higher than observed R&D intensities.

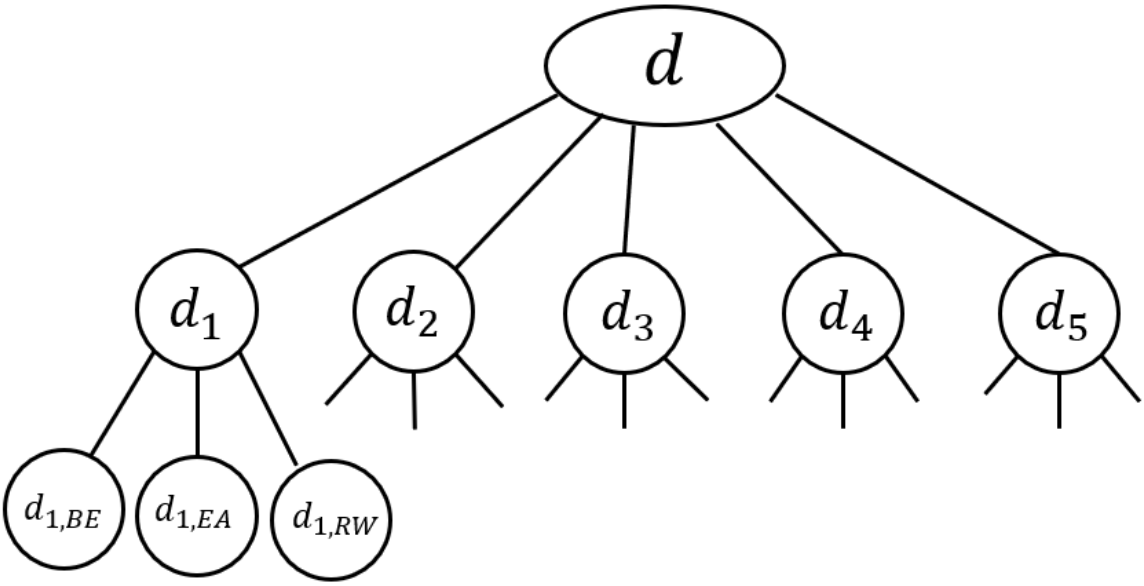
In the model, the solution to this problem is based on the interpretation of A as the stock of intentional R&D efforts. It is probable that there are other forms of knowledge that are also relevant for production but that are not the product of intentional R&D activities. We therefore have included a "general technology" term GT to the model that is calibrated on the gap between the observed labour income share and $\frac{r_A}{\gamma_A+\delta_A}$ times the observed R&D intensity. The modified production function then becomes:

$$Y = K^{(1-\sigma^M)(1-\alpha)} \left(\frac{A+GT}{1+\gamma_A} L\right)^{(1-\sigma^M)\,\alpha} M^{\sigma^M}$$

and the FOC associated with R&D capital:

$$r_A \, PRD \, \frac{A+GT}{1+\gamma_A} = (1-\sigma^M)\,\alpha\, MC\, Y$$

There are, moreover, $2n+3$ demand aggregates per region in the model: private consumption, government consumption and investment, plus an intermediate consumption and investment good per industry. Each good is assumed to be a CES aggregate of n inputs from the different industries, each of which is itself an aggregate of regional varieties. The double layered structure is represented by the scheme below, for 5 industries. d is the aggregate demand category, d_i signifies the component from industry i and $d_{i,r}$ is the variety of region r in component i. As the calibration section will point out, components in the upper layer are more complementary in nature at this broad level of aggregation (i.e., the elasticity of substitution is below 1), whereas the regional varieties can be rather thought of as substitutes for one another (i.e., the elasticities of substitution exceed 1).



The government consists of both a fiscal and a monetary part. The fiscal government levies product taxes as well as income taxes on labour and capital, and pays for social benefits, public consumption and public investment, gives a tax credit for private investment, subsidizes the wages for researchers and issues bonds at the nominal interest rate, which are held by the households. A lump-sum tax rule ensures that the public debt rate meets a long-term target. This rule guarantees that any imbalance in government finances due to an exogenous shock does not result in an exploding debt (either to plus or minus infinity) but is translated into a lump-sum tax variable, that symbolizes the budgetary sustainability of the shock in a stationary way, without the distortions that come with other forms of taxation. The government spends a fixed percentage of GDP on public consumption and investment, the latter accumulating into public capital that increases the productivity of the private sector. Finally, a Taylor rule, representing the central bank policy, sets the nominal interest rate as a lagged function of the deviation of inflation to its long-term target and the output gap. Belgium and the rest of the Eurozone share the same Taylor rule, representing the policy by the European Central Bank.

As already mentioned before, there are 3 regions in the model: Belgium (BE), the rest of the Eurozone (EA) and the rest of the world (RW). Each region contains the same type of equations for the household, production, R&D and government sectors. Each demand aggregate contains a trade-off between regional varieties of each industry's product. Trade balances are accumulated into foreign debt positions. There are two foreign debt stocks, both held by Belgian households: one with respect to EA and one with respect to RW. The debt holdings between EA and RW do not constitute an independent variable because they can be offset by the Belgian EA and RW debt holdings. There is a risk premium associated with foreign debt that ensures that the debt-to-GDP ratios return to a constant in steady state. Trade between RW and the Eurozone is modulated by an exchange rate variable. Due to monetary union, the exchange rate between BE and EA is fixed to 1, but in the process of making the model stationary, the fact that all prices in BE and EA are expressed in terms of the consumer price deflator, a variable naturally arises in equations describing international transactions or inflation that converts a BE price into an EA price and that de facto acts as an exchange rate. Like the exchange rate, this proportion of the BE with respect to the EA consumer price deflator is stationary. Through this variable, a kind of "internal devaluation" is possible in the model using the difference in tastes between the BE and EA households as a wedge.

Resource constraints impose that for every industry, output equals total demand from all sources, at least in the steady state. But these constraints must also be consistent with the income constraints in the model. Indeed, substituting the firms' profit functions, the trade balance and the government's budget constraints into the household income constraint equates total income to total expenditure in each region, but total expenditure may also include adjustment costs. The resource constraints are modified so that adjustment costs drive a wedge between income and expenditures on the adjustment path, as they are supposed to do.

A number of adjustment costs have been included in the model. Price rigidity is introduced by inserting a convex price adjustment cost, or more specifically, a quadratic Rotemberg (1982) cost, in the profit function of the firms. This implies that profit maximization becomes an intertemporal problem. The resulting FOC defines the price-wage mark-up in steady state as a constant depending on the industry-specific elasticity of substitution but allows prices to move slower than marginal costs in between equilibria. Wage, labour and investment adjustment costs are also introduced through a Rotemberg (1982) specification. Also, there is a quadratic cost associated with deviations from the steady state utilization rate of non-R&D capital.

3. Calibration

3.1. IO tables

3.1.1. General methodology

The model's input-output (IO) block is based on the FIGARO inter-country symmetric product-by-product IO tables for 2015 provided by Eurostat, which contains data for 64 products (a.k.a. homogeneous industries). The methodology underlying the FIGARO tables[7] corresponds to the 'European Systems of Accounts 2010' (ESA 2010). All matrices described in this section are in basic prices, where trade and transport margins have been included in the products of aggregates G ("Wholesale and retail trade") and H ("Transportation and storage") whenever applicable.

Countries are grouped into 3 geographical areas: Belgium, the rest of the Euro Area (henceforth EA) and the rest of the world (RW), cf. the scheme below. The 64 products can be lumped into an exercise-specific number n product categories. The resulting matrix's columns then contain $2n + 3$ demand aggregates per region: both an intermediate and an investment good used by each industry, as well as private consumption, government consumption and government investment. The $3n$ rows represent the products from each region used by the different demand aggregates.

To increase consistency with official National Accounts (NA) figures[8], the Belgian domestic part of the matrix (BE Domestic on the scheme) is replaced by a symmetric product-by-product IO table derived from the official Belgian product-by-industry matrix for 2015 by applying the industry technology assumption (IT), i.e., by distributing a heterogeneous industry's inputs proportionally[9] over the products it creates. This method was preferred over directly taking the official symmetric table for reasons of both transparency and consistency with other data. Indeed, many NA variables like labour market characteristics, investments or product taxes are only available per industry and therefore have to be converted into per product terms. Grosso modo, there are 2 ways of doing this: IT described above and the commodity technology (CT) assumption, which assumes that each product is created using the same mix of inputs regardless of the industry in which it is produced. Mathematically, the first method can be described as calculating the matrix $X_{IT} = \mathcal{U}\, V'$ where $\mathcal{U}$ is the use matrix and V is the supply matrix expressed in row proportions, while the second method consists in finding $X_{CT} = \mathcal{U}\, W^{-1}$ (assuming that the number of products and industries are equal) where W is the supply matrix in column proportions. At first sight, the CT approach may seem the less crude and more intuitive method, though this impression may be based mainly on examples of very disaggregated product categories. Its disadvantage is that the inverse matrix in the calculation does not preclude X_{CT} from containing negative values when the values in $\mathcal{U}$ and W are all positive, as is the case under IT. Indeed, $\mathcal{U}\, W^{-1}$ was found to contain a lot of negatives for the base year in Belgium, which provides some evidence that the CT assumption is

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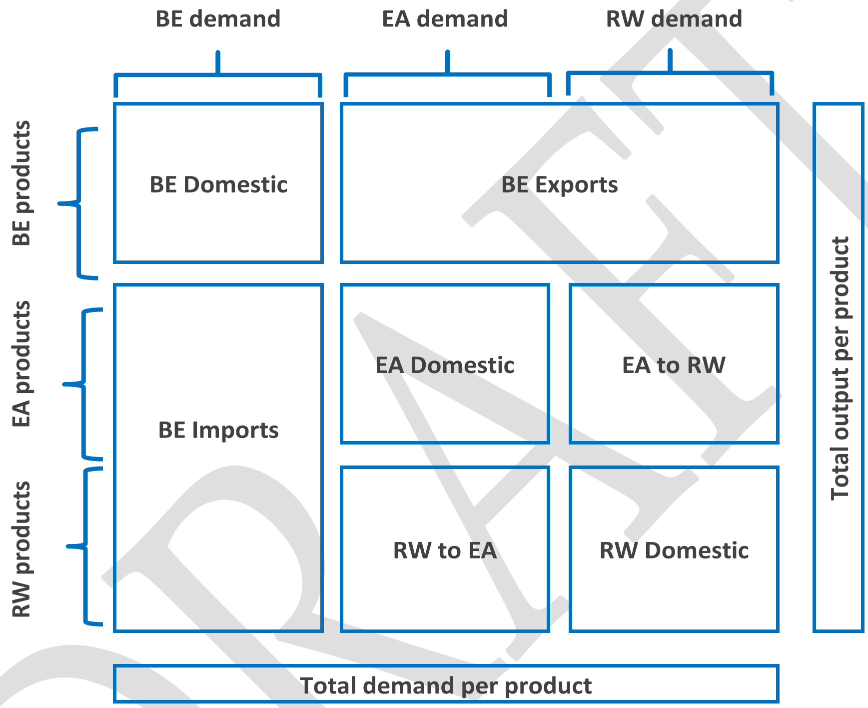
7 Detailed information on the methodology can be found here: https://ec.europa.eu/eurostat/web/esa-supply-use-input-tables/methodology

8 For instance, there is a difference of 15% between the official NA and FIGARO table for investment (P51G) in the product R&D (M72), which is crucial for this exercise.

9 More specifically, proportionally to each product's share in the industry's total output, which can be read from the supply table.

probably not very appropriate on this level of aggregation. To correct for this, official product-by-product tables often make ad hoc adjustments, which are not very easy to trace. However, an implicit conversion matrix $\mathcal{C}$ can be calculated from the officially published use and product-by-product matrices: $X = \mathcal{U}\,\mathcal{C}$. In principle, this matrix could be used to express other per industry variables in per product terms consistently with the official symmetric matrix. But unfortunately, this matrix too contains many negatives, and applying it to per industry labour market and other variables often generates negative values. Therefore, IT seemed the most reasonable option for this exercise. It respects observed industry data but remains agnostic about how an industry's inputs are distributed over its products. Given that it was applied to all industry variables, it was also used, for consistency, to convert the use matrix in a symmetric product-by-product matrix, different from the official one.



Analogously, Belgian exports and imports are replaced by symmetrized official NA tables, while FIGARO data are used as keys to distribute the aggregates over destination countries and products for exports and over countries of origin for imports, information that is lacking from the official tables.

3.1.2. Investment

The official NA and FIGARO IO tables only contain the product composition of aggregate investment. For the model, this column has to be split into a symmetric product-by-product table for private investment and a column (also by product) for government investment.

For Belgium, we disposed of detailed product-by-industry investment matrices per institutional sector, implying we could break down the investment column into per industry investment and government investment. As our data did not contain information about the country of origin of products, we applied the weights of aggregate investment to distinguish between domestic and imported goods for each industry and government investment. Afterwards, the domestic and import tables were symmetrized following the IT method described above. The tables were then transformed into row proportions, i.e., expressed in terms of shares of the aggregate investment column. In a next step, the domestic table weights were applied to the aggregate investment products originating from Belgium, while the import table weights were applied to all foreign countries.

For other countries in the Eurozone and the rest of the world, we made use of Eurostat and OECD data on (1) aggregate gross fixed capital formation (GFCF) split into 10 different asset categories and (2) cross-tables of GFCF by asset and industry (for 64 industries). Missing data were treated in a two-step procedure. Firstly, missing values in the breakdown of aggregate GFCF by asset were imputed using weights from other countries (in the same group) with complete information, assuming statistical independence. Secondly, this information could then be included into the cross-table, so that at least total GFCF by asset and total GFCF by industry were known. Whenever GFCG for a particular asset in a particular industry was missing, the statistical independence assumption could be invoked to assign a value to this cell, of course taking into account non-missing values and column and row totals. Detailed calculations are available on request.

Government investment had to be distilled out of the resulting matrices. We used information from Eurostat and the OECD about the stocks of capital by asset and by institutional sector. We approximate the proportion of government investment in total investment by the proportion of government capital in the total capital stock, which is expected in the long term, assuming that depreciation rates between private and public capital are equal within asset types. These proportions can then be used to calculate government investment per asset type, which are reweighted to match observed total government investment. To avoid double counting, government investment should be subtracted from the product by industry matrices derived above. We applied the rule that government investment is subtracted from industry O ("Public administration and defence, compulsory social security") and, for assets where the investment by O is not sufficient, the rest is subtracted from industries P ("Education") and Q ("Human health and social work activities") proportionally.

The resulting matrices are in an asset-by-industry (including government) format. The IT assumption described above was used to convert them into asset-by-product matrices. We then devised an assets-to-products correspondence methodology. Details can be provided upon request, but for R&D the correspondence was 1-to-1: product M72 was identified with asset N1171G. The next steps to break down the aggregate investment column in the FIGARO matrix are identical to the method for Belgium. A generic EA and RW country were constructed as a weighted average of the countries with enough information and applied to those countries that did not have enough information.

3.2. Tax variables

3.2.1. Product taxes

For Belgium, detailed taxes are available on the same detailed level as the use matrix, i.e., for 64 products used in 64 industries (as intermediate consumption), private and government consumption and investment. Taxes are distributed between domestic and imported goods proportionally to expenditures, except for the category of "import duties", which is assumed to be only levied on import goods, and product subsidies, which are assumed to be only attributed to domestic goods. For investment goods, the same level of detail for the value added taxes has been observed, while other product taxes are broken down according to detailed investment expenditures so that each industry in effect pays the same tax rate per product.

For the Eurozone and the rest of the world, the (64x64) product-by-industry tables "Taxes less subsidies on products" from Eurostat and the OECD have been used, together with the corresponding use tables (as tax base). Net taxes and their base were first symmetrized by country following the IT method, then aggregated per region (EA and RW) and product. Finally, tax rates were calculated by dividing taxes by their base. For Germany and Spain, product taxes were not available on a detailed level, so their taxes were calculated using weighted tax rates of other European countries and reweighted to match the total tax receipt.

3.2.2. Other tax variables

Labour and capital income taxes, as well as consumption taxes, benefit replacement rates and tax credits for R&D investment, were taken over from the Belgian version of QUEST III R&D. For Belgium, we updated labour and capital income tax rates and consumption tax rates following the (implicit tax rate) methodology of the Commission's Directorate-General Taxation and Customs Union (DG TAXUD), as described in the recurring publication "Taxation Trends in the European Union" (DG TAXUD, 2020). For Belgium, we also added information on the wage subsidy rate for researchers, based on internal industry-specific information (which was converted in per-product terms assuming IT) and revised the tax credit on R&D investment, based on information on fiscal expenditures.

3.3. Other variables

3.3.1. Labour market variables

Besides employment per industry and skill level, we also need R&D employment per industry. The latter is often only available in number of persons, not in hours worked. For consistency, we also took employment per industry in numbers of persons, from Eurostat and OECD data. Combining these data with the distribution of skill levels among the employed per industry and country from the KLEMS labour accounts, we could then derive the number of low-, medium- and high-skilled per industry and country. Applying Eurostat and OECD data on skill-specific unemployment rates (which, unfortunately, are not industry-specific) allowed us to calculate the number of unemployed people and the total labour force per industry and skill level.

The number of R&D workers was derived also from Eurostat and OECD data. Missing values for a country were imputed using information from other years using the rule of three. The number of R&D workers was then subtracted from the number of high-skilled in the respective industry.

Labour income shares were derived from the compensation of employees and total output lines in the symmetric FIGARO table and the Belgian use table. For Belgium, these 2 variables were symmetrized in the usual way. Both variables are aggregated per region and product, after which the tax rates are obtained by division.

Relative wages between skill levels were taken from the OECD's Education at a Glance series. These values are not industry-specific, so the aggregate relative wages are used for each industry.

3.3.2. R&D variables

R&D intensities were derived from the OECD's Business Expenditure on R&D (BERD) indicator and total output. Because BERD data contain many missing data, we first tried to impute missing values for a country in 2015 using information from other years. Remaining blanks were filled in using information of other countries in the same region. Imputation was done always by using the rule of three and respecting observed values. Countries with practically no information were excluded from the analysis. In a next step, the BERD and total output variables were again symmetrized, aggregated and the former divided by the latter to get R&D intensity per region and product.

The research output elasticity of R&D workers in an industry is a parameter constrained by the model and by observables, to wit: the R&D intensity, the labour income share, employment of researchers and the 3 skill levels in production and the relative wages between skill levels.

Spillover elasticities of Belgian stocks of knowledge to other domestic and foreign stocks were calibrated on the innovation network composed by Liu & Ma (2022), that the authors kindly provided to us. Note that the sum of the elasticities per industry is constrained by the semi-endogenous growth hypothesis and equals $\sum_{k=1}^{n} \varphi_{ik} = 1 - \lambda_i^{RD} \frac{\log(1+npop)}{\log(1+\gamma_A)}$, where λ_i^{RD} is the research output elasticity and $npop$ and γ_A are the population and technology growth rates, respectively. For the EA and RW stocks of knowledge, we assume in this exercise that they are not affected by changes in Belgian knowledge stocks and move exogenously.

3.3.3. Mark-ups

Mark-ups by industry were estimated using the methodology described by Roeger (1995), who shows that under constant returns to scale, absence of factor hoarding and imperfect competition on the product market, the price-cost mark-up can be estimated by regressing the difference of the primal and dual Solow residuals on the difference between value added and the compensation of capital. We adapted the method slightly to take intermediate consumption into account and applied the estimation procedure developed by Christopoulou and Vermeulen (2008), inter alia by using the same user cost of capital concept to construct an ex-ante measure of capital compensation, on the EUKLEMS data on the evolution of total output, labour compensation, intermediate consumption and capital services for European countries and the US.

3.4. Parameters

As for the Frisch labour supply elasticity, a large interval of estimated values exists in the literature, going from close to 0 in micro-estimations (see e.g., Bargain et al., 2011; Chetty et al., 2011; Keane, 2021; Reichling & Whalen; 2015) to 2 in macro-estimations (see e.g., Smets & Wouters, 2007; Linde et al., 2016; Albonico et al., 2019). We chose to follow results from Keane & Wasi (2016), who conducted a micro-study taking human capital, lifetime labour supply and the difference between the intensive and extensive margins into account and arrived at skill-specific estimates of 1.04 for low-skilled workers, 0.71 for the medium-skilled and 0.5 for the high-skilled. These results lie in the line of other intermediate estimations, like those of Browing et al. (1999) and Fernandez-Villaverde & Rubio-Ramirez (2007). They are also close, on average, to the value used in QUEST III R&D, 0.72.

A second important labour market parameter is the substitution elasticity between labour skills in the production function. For the rest of the world, we take a consensus value in the literature of 1.6, as reported in to Jerzmanowski and Tamura (2020). For Belgium, we take the average of the interval of estimates that they obtain using data on 32 countries, one of which is Belgium: 2.1. The value for the Eurozone is set equal to the latter.

The output elasticity of public capital is set to 0.12, which is the long-term estimate found in the meta-study by Bom & Ligthart (2014). See also Biatour et al. (2017) for a justification of this value for Belgium.

As for the substitution elasticity between different products, we take a value of 0.5 for all regions and demand aggregates. The empirical study of Atalay (2017) comes up with a wide interval for this elasticity in different countries, although all values are below 1, suggesting that products, at least at a fairly high level of aggregation, are mainly complementary to each other. We take a central value of 0.5, as in Bergholt (2014). Estimates of the substitution elasticities between regional varieties of a certain product are mostly larger than 1, indicating that they mainly act as substitutes. For Belgium, we take the value of 1.43 estimated by Aspalter et al. (2016), while for the Eurozone and the rest of the world we take the value of 1.5 that is commonly used in macroeconomic models.

As for the dynamic adjustment parameters, we have chosen values from other macroeconomic models who applied the same specification for the associated cost function as the one in our model. We took a value of 19.745 for the price adjustment cost, of 120 for the wage adjustment cost and of 0.05 for the capital utilization adjustment cost, as in QUEST III R&D. The labour adjustment cost was fixed provisionally at 250 for all skill levels, a value of the same order of magnitude as most of the skill-specific values in QUEST III R&D. The investment adjustment cost parameter was taken to be 7.7, as in De Walque et al. (2017).

In the Taylor rule describing the monetary authority's behaviour, we fix the persistence parameter at 0.85, as in Kollmann et al. (2016), the sensitivity of the interest rate to inflation at 1.5 and to the output gap at 0.05, close to values reported by the literature (see e.g., Kollmann et al., 2016; Giovannini et al., 2019; De Walcque et al., 2017), for both the Eurozone and the rest of the world. The debt correcting tax rule will include a sensitivity of 0.003 with respect to the debt target and of 0.023 with respect to the deficit, an average of the values found by Albonico et al. (2019) for European countries.

4. Results

This section first discusses the results of the calibration. Next, for a disaggregation of the economy into 8 industries, a rise in the wage subsidy rate for researchers of 10% is simulated for each of the first 7 industries[10] separately. The long-term effects on GDP of each reform is computed, as well as a “bang for the buck” measure to compensate for the fact that the budgetary cost of the different reforms is not the same. Finally, the simulation with the highest bang for the buck is discussed in more detail.

4.1. Calibration results

The model was calibrated for 8 industries, that are described in table 1 below. Industry 1 groups mostly low-tech manufacturing, utilities and construction, while industries 2-6 contain medium- to high-tech manufacturing sectors. Industries 7 and 8 consist of market and non-market services, respectively.

Table 1 : Code and description for 8 model industries

Industry	NACE rev. 2 codes	Description
1	A-F (without sectors 2-6 below)	Agriculture, Mining, Low-tech manu-facturing, Utilities, Construction
2	C20-21	Chemicals, Pharmaceuticals
3	C22	Rubber and plastics
4	C26-27	Electronics, Electrical equipment
5	C28	Machinery and equipment
6	C29-30	Transport equipment
7	G-N	Market services
8	O-U	Non-market services

Table 2 displays some key characteristics of these 8 industries in the Belgian economy in 2015. Services account for 65% of total output and 79% of total value added. The low-tech manufacturing aggregate creates 25% of total output and 15% of value added. Only 10% of total output and 6% of value added is created by medium- to high-tech manufacturing industries. The relatively higher share of services in

[10] There was no simulation for the 8th industry, non-market services, because of a seemingly different methodology for treating public R&D between our model on the one hand and the spillover data of Verspagen (1997) and Liu & Ma (2022) on the other. In our model, public R&D is part of public investment, accumulating into public capital, which affects private productivity in the same way as, e.g., infrastructure. This is inconsistent with the two aforementioned studies, where citations to patents arising from public R&D are counted as citations to the sector of non-market services.

value added reflects of course the fact that their intermediate consumption share is lower than in manufacturing sectors.

Mark-ups (column 4) are highest in market services, and relatively high in chemicals & pharmaceuticals as well as in machinery & equipment. The sector of transport equipment has the lowest economic profits. The share of costs in total output (column 5) is just the inverse of the mark-up. This share should cover the compensation of labour (column 7) and capital, as well as the purchase of intermediate goods (column 6). However, as is clear from column 8 of the table, the share that is left from the total costs after subtracting the observed labour income and intermediate share, is negative for some industries, indicating that there can be no normal compensation for capital. This is inconsistent with the positive observed investment shares for these industries as shown in column 9. Three possible explanations for this divergence come to mind. Firstly, it may be an indication of profit shifting. All involved sectors (chemicals & pharmaceuticals, machinery & equipment and transport equipment) contain large multinational enterprises that may "hide" part of their capital compensation as intermediate consumption. Secondly, the labour income share may be inflated because labour was able (e.g., through wage bargaining) to capture a part of economic profits. The method of Roeger (1995) to estimate mark-ups assumes competitive input markets, so using the observed income share in the presence of labour bargaining power will surely distort results. A solution may lie in adapting the method to explicitly account for bargaining power and the distribution of economic profits between labour and capital. Thirdly, table 2's eighth column is to be interpreted in terms of a capital user cost only as, in the model's current form, households optimize their capital holdings with respect to "normal" income, i.e., the compensation of capital disregarding economic profits. The total compensation of capital, however, also includes (part of) economic profits, which are more than sufficient to make up for "losses" in the user cost part.

Table 2 : General industry characteristics

Industry	Share in total output	Share in total VA	Price-wage mark-up	Share of costs in total output	Intermediate share	Labour income share	Rest	Investment share
1	0.254	0.150	1.12	0.89	0.74	0.14	0.007	0.06
2	0.054	0.034	1.20	0.83	0.73	0.11	-0.005	0.13
3	0.008	0.005	1.11	0.90	0.72	0.19	0.000	0.06
4	0.007	0.006	1.15	0.87	0.64	0.21	0.015	0.10
5	0.011	0.008	1.20	0.84	0.65	0.20	-0.011	0.06
6	0.018	0.007	1.06	0.95	0.84	0.13	-0.023	0.04
7	0.500	0.563	1.29	0.78	0.51	0.22	0.052	0.13
8	0.149	0.227	1.10	0.91	0.33	0.57	0.003	0.04

Reflecting the distribution of aggregate value added over industries, the total labour force is concentrated in service sectors, which account for more than 80% of workers, whereas low-tech manufacturing has a share of 15% and medium- to high-tech sectors a share of only 4%, as can be seen from the second column in table 3. As for the breakdown of skill levels per industry, the share of low-skilled workers (column 3) is highest in low-tech manufacturing and lowest in chemicals & pharmaceuticals and non-market services. The distribution of high-skilled workers (column 5) presents a mirror image of this, with relatively higher concentrations in non-market services and chemicals & pharmaceuticals and a relatively lower concentration in low-tech manufacturing. The share of private sector researchers, however, does not follow the same distribution as the high skilled. As column 8 shows, in the sector of electronics and electrical equipment, a higher fraction of the high-skilled are researchers, so that this sector also have the highest proportion of researchers, whereas in non-market services, almost no private sector researchers are employed.

Table 3 : Labour market characteristics per industry

Industry	Share in total labour force	Share of low-skilled	Share of medium-skilled	Share of high-skilled	Share of (private) researchers in total high-skilled employment
1	0.154	0.31	0.50	0.19	0.035
2	0.014	0.13	0.38	0.48	0.158
3	0.005	0.28	0.50	0.22	0.084
4	0.006	0.21	0.43	0.36	0.334
5	0.007	0.21	0.51	0.28	0.141
6	0.008	0.26	0.54	0.21	0.135
7	0.462	0.22	0.41	0.37	0.025
8	0.344	0.15	0.32	0.52	0.000

Note that only active workers per industry are observed in Eurostat data. The share of different skill levels per industry were taken from the EUKLEMS data and used to calculate the number of workers per skill level. Then, from Eurostat data, overall skill-specific unemployment rates, shown in table 4, were used to estimate the number of unemployed per skill level and industry, so that the labour force per industry and skill level could be constructed.

Table 4 : unemployment rates per skill level

Low-skilled	Medium-skilled	High-skilled
0.147	0.075	0.041

Finally, table 5 describes the most important R&D related variables per industry. The R&D intensity (column 2) is highest in electronics & electrical equipment, confirming information on the share of researchers. Chemicals & pharmaceuticals and Machinery & equipment also have high R&D intensities. Column 3 contains the output elasticities of researchers per industry, which are constrained by model equations and observations on R&D intensities, labour income shares, employment per skill level and of researchers, and relative wages between skill levels. The values obtained mostly fit very well in the [0,0.50] interval for estimates based on sectoral data mentioned by the Congressional Budget Office (2005).

Table 5 : R&D characteristics per industry

Industry	R&D intensity	Implied R&D output elasticities	Share of researchers in industry labour force	Share of researchers in total labour force	Wage subsidy rate (%)
1	0.004	0.38	0.03	0.005	0.42
2	0.033	0.36	0.15	0.002	1.58
3	0.015	0.35	0.08	0.000	0.65
4	0.074	0.55	0.32	0.002	2.24
5	0.029	0.41	0.14	0.001	1.12
6	0.016	0.38	0.13	0.001	0.65
7	0.009	0.32	0.02	0.011	1.18
8	0.000	0.29	0.00	0.000	1.52

Column 4 presents the share of researchers in each sector's labour force and is the multiplication of the last two columns in table 3, whereas column 4 shows the share in the total labour force, thereby giving an indication of the relative size of the R&D labour force between industries. Clearly, despite employing less researchers in relative terms, the sectors of market services and of low-tech manufacturing still employ a majority of researchers in absolute numbers. The last column in the table contains an implicit industry-specific subsidy rate to the wage of researchers, based on in-house information about the fiscal expenditures on the subsidy and about the salary mass of researchers per industry.

The matrix of spillover coefficients is, as mentioned before, based on patent citation information collected by Liu & Ma (2022). Figure 1 contains a heat map for patent citations made by the 8 Belgian industries (in rows), broken down with respect to the industries to which the cited patent belongs (columns). The first 8 columns contain the Belgian, the middle 8 columns the EA and the last 8 columns the RW industries of origin. The three "main diagonals", representing own-industry spillovers, are darker and hence more important. Also, the rest of the world is more important to Belgian knowledge creation than Belgian or even Eurozone Figure 1 : heat map of patent citations (Liu & Ma, 2022)knowledge stocks, as is indicated by the fact that the right, RW, panel in figure 1 is darker than the left and middle panels. Finally, within the RW panel, industries 4 (electronics & electrical equipment) and 7 (market services) are more often used as source for knowledge creation in other sectors, which is shown by darker corresponding columns.

Figure 1 : heat map of patent citations (Liu & Ma, 2022)

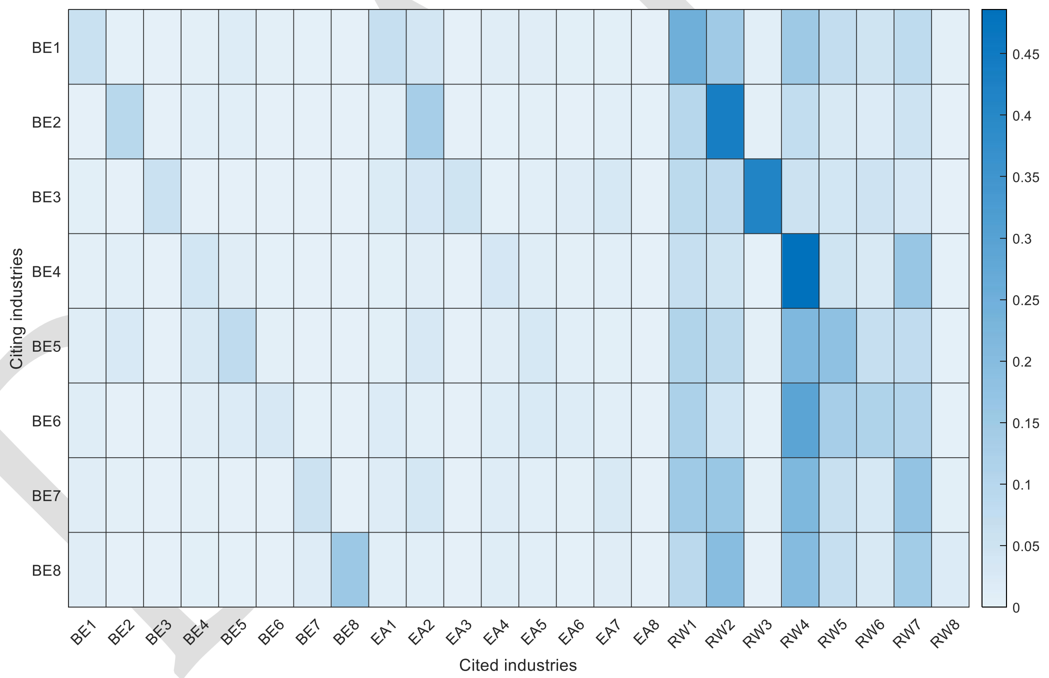
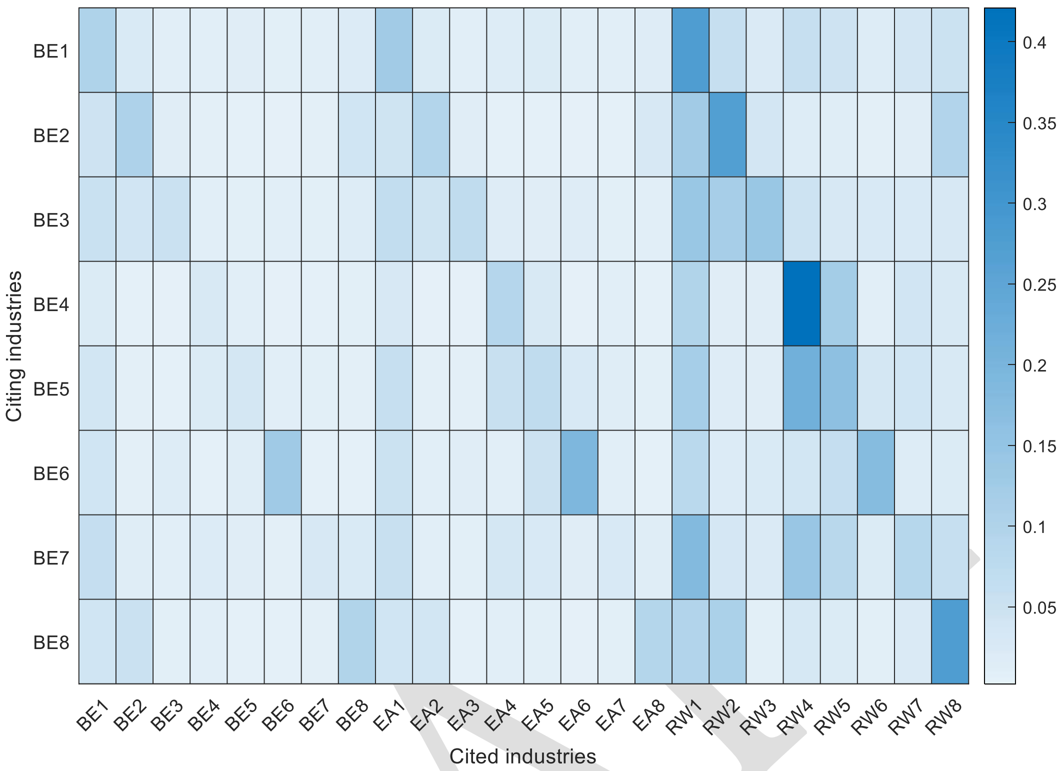


Figure 2 creates a similar heat map for patent citations collected by Verspagen (1997). Grosso modo, the same picture emerges, confirming the finding by Acemoglu et al. (2016) that the innovation network is persistent. However, one clear deviation apparent in these earlier data is the fact that industry 1 (low-tech manufacturing etc.) plays a more central role as the source for other sectors' knowledge creation, as shown by the darker columns associated to this industry (for RW and, to a lesser extent, for BE and EA).

Figure 2 : heat map of patent citations (Verspagen, 1997)



4.2. Simulation results

As stated before, the 7 market sectors were, each in turn, given the same 10 percent point shock of the subsidy rate of researchers' wages, to find out which shock would generate the largest effect on long-term GDP. Table 6 contains the answer to this question. We use the more recent spillover data gathered by Liu & Ma (2022) in our simulations but add the results using the Verspagen spillover matrix as a measure of sensitivity. Clearly, the subsidy shock to the sector of market services has the largest long-term effect on GDP, of 1.59%, followed by the shock to the sector of low-tech manufacturing, resulting in an increase of 0.67%. All other shocks have effects on a smaller order of magnitude. Though the results using the Verspagen matrix change the exact numbers, the relative order between the shocks is preserved.

Table 6 : Long-term effects on GDP of 10ppt shock in R&D wage subsidy rate per industry

Industry	Long term change in GDP (%) Liu-Ma	Long term change in GDP (%) Verspagen
1	0.67	0.80
2	0.04	0.09
3	0.01	0.04
4	0.03	0.07
5	0.03	0.04
6	0.02	0.04
7	1.59	1.56

Note, however, that the number of researchers and their wages may differ between industries, so that these 7 shocks do not have the same budgetary cost. To account for this fact, the third column in table 7 contains the long-term cost, ex post - so including behavioural changes - as a % of ex post GDP. Note that the second column of table 7 differs slightly from the second column of table 6 because the former is expressed as a % of ex post GDP so as to enable a consistent comparison with the budgetary cost. Apparently, the reform in sector 7, that had the largest effect on GDP, also has a much larger cost.

Table 7 : Long-term efficiency of 10ppt shock in R&D wage subsidy rate per industry

Industry	Long term change in GDP (%)	Ex post cost in % of GDP	"Bang for the buck"
1	0.67	0.0088	76.0
2	0.04	0.0168	2.6
3	0.01	0.0011	12.7
4	0.03	0.0077	4.2
5	0.03	0.0032	9.0
6	0.02	0.0027	7.3
7	1.57	0.0353	44.4

As a simple efficiency measure, or “bang for the buck”, the fourth column of table 7 contains the ratio of the long-term yield and cost of each shock. The ratio is maximized for industry 1, low-tech manufacturing etc., indicating that an additional wage subsidy for researchers would generate the largest results when concentrated on this sector. A wage shock to industry 7, market services, would have an efficiency in the same order of magnitude. By contrast, wage shocks to high-tech industries like chemicals & pharmaceuticals or electronics & electrical equipment have a much smaller efficiency, reflecting both the smaller centrality of these industries in the innovation network and a higher saturation effect relative to other industries.

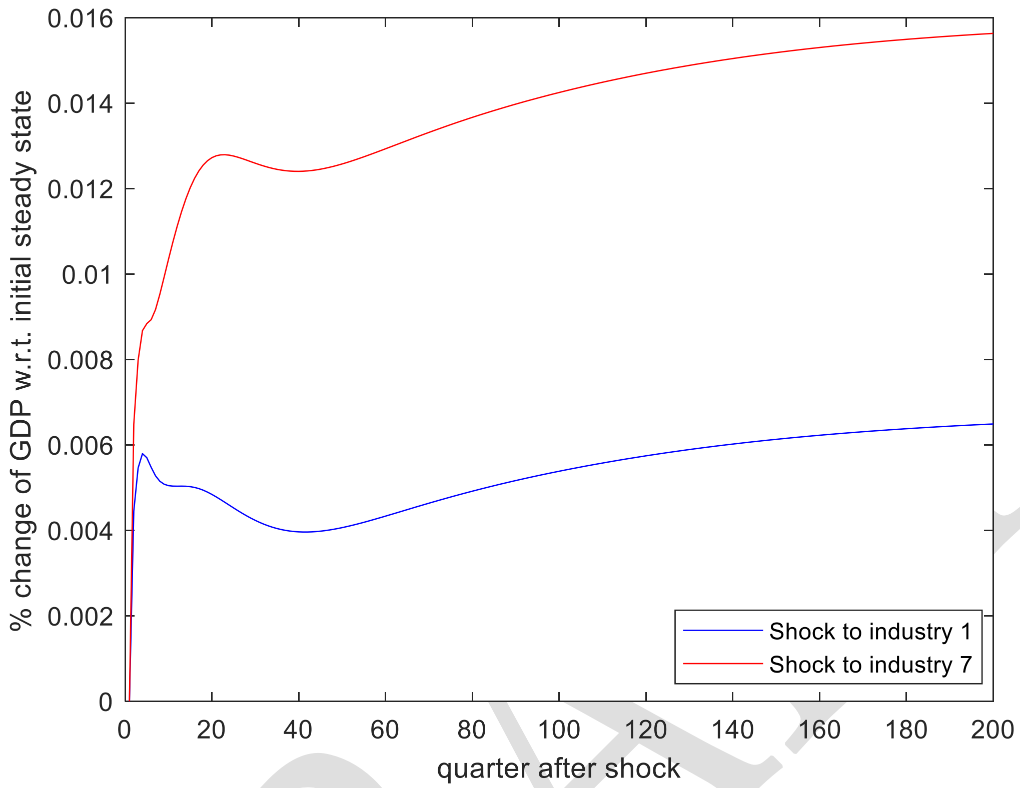
Table 8 contains the long-term effects of the 7 shocks on some other variables of interest. All shocks result in a similar relative increase in the number of researchers in the respective industry, as is shown in the second column. In all industries, this results in a decrease in the number of high-skilled workers in production (column 3). The change in real output (i.e., output divided by the consumer price deflator) of the industry itself follows the distribution of the long-term effect on overall value added, the change being the largest for sector 7. The change in output quantities (i.e., output divided by the industry production price deflator) is higher than the change in real output in every case, reflecting the fact that the shock provokes a productivity increase, which lowers the price of the industry product.

Table 8 : Effect of 10ppt shock in R&D wage subsidy rate per industry on other variables of interest

Industry	Change in number of researchers in sector (%)	Change in high-skilled labour in production (%)	Change in real output (%)	Change in output quantity (%)
1	10.98	-0.29	0.70	1.58
2	10.26	-1.29	0.16	0.58
3	10.77	-0.67	0.22	0.95
4	9.29	-2.97	0.36	1.32
5	10.48	-1.15	0.31	1.15
6	10.47	-1.15	0.23	0.75
7	11.05	-0.25	1.51	2.16

Taking only long-term effects into account, increasing R&D wage subsidies in industry 1 seems preferable. However, transitional dynamics may still shift this balance due to more or less desirable temporary effects. For instance, figure 3 compares the evolution of Belgian GDP over time for the shocks to industries 1 and 7, which have the highest bang for the buck score in the long run.

Figure 3 : Effects on GDP of an R&D wage subsidy shock in industries 1 and 7 over time



Note that the difference in long-term GDP is, as already explained, a function of the different budgetary size of the two measures. Under both scenarios, an initial sharp increase in Belgian GDP is followed by a contraction, before convergence to the long-term GDP level sets in. In both cases, the initial rise is supported by wage rigidity keeping the wage below increasing labour productivity, but in the medium run, inflationary pressures in Belgium due to increased demand push nominal wages further up and start to adversely affect the relative competitivity with respect to the Eurozone (relative to a long-term rise because of a positive technology shock), as is shown in figure 4. In the first scenario, which is the most preferable of the two in the long run, this contraction is more pronounced and lasts longer because of the relatively larger part of intermediate consumption in industry 1, which is largely composed of manufacturing industries, and a greater importance of imports therein.

This temporary slowdown also has some adverse effects on the labour market, as figure 5 shows for industry 1 in the first scenario. Indeed, not only are employment gains unequally distributed among skill levels, but the employment of low- and medium-skilled workers is even lower than the initial steady state level in the medium run. While this effect is very small due to the limited size of the shock in this scenario, in principle the characteristics of the transitional path could change which scenario is preferred from a policy perspective.

Figure 4 : Evolution of wage cost in BE relative to EA (ratio 1 in initial steady state)

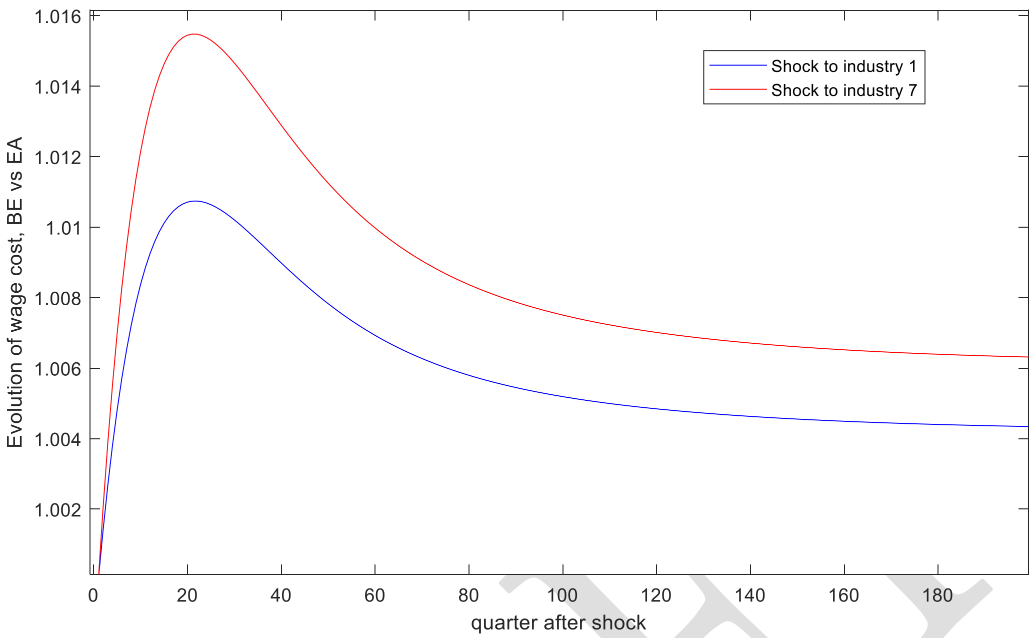
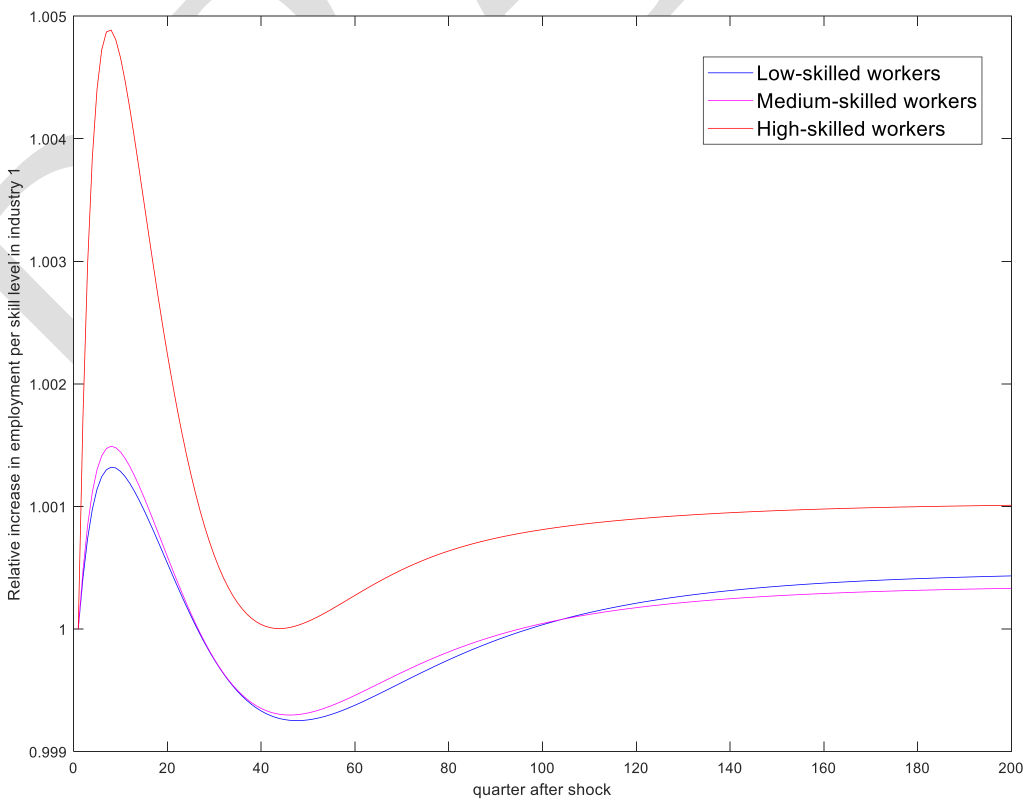


Figure 5 : Relative increase in employment per skill level in industry 1 (after shock to industry 1)



5. Conclusion

This paper explores how the existence of an asymmetric innovation network between industries may influence in which industry a marginal increase in the wage subsidy for researchers may be the most beneficial. To that end, a multisectoral dynamic general equilibrium model with semi-endogenous growth and industry-specific stocks of knowledge is developed and calibrated, where spillovers between the latter are based on the information about patent citations compiled by Liu & Ma (2022).

Our preliminary results seem to support the idea that the best locus for an increase in subsidies may not be the industries most conspicuous when it comes to innovation, but rather industries that occupy a central role in the innovation network, in the sense that they trigger additional innovation in other sectors. Although the ranking of sectors according to long-term efficiency of support follows their centrality in the innovation network, short- to medium-effects caused by demand factors and adjustment costs along the transition path may alter the relative desirability of policy alternatives.

The work presented here can be improved in several ways. Firstly, we plan a more refined analysis of the importance of industries now aggregated in the first sector, which turned out to be the most preferable candidate for additional subsidies, at least from a long-term perspective. Secondly, rather than taking fractions of patent citations as a proxy for technological spillovers, we would like to incorporate direct estimations of industry-specific spillovers, which can take into account asymmetries between industries. Thirdly, acknowledging the existence of labour bargaining power in the estimation of markups will allow for identifying the "true" labour income share during the calibration of the model and solve the problem of negative capital income shares in some industries.

More fundamentally, the assumption of equal long-term growth rates across sectors may be unwarranted as is argued in, e.g., Baumol (1967). The model could be adapted to account for this, by splitting each demand category into a Cobb-Douglas aggregate of a "Baumol" and a "non-Baumol" good, where the lower growth rate of the former would be compensated by a higher rate of deflation. Such an adaptation, however, would entail severe constraints on model parameters to guarantee balanced growth (in nominal terms).

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7. Technical appendix: stationary model equations

The technical appendix in section 8 contains the definition of symbols for variables and parameters used here.

There are three regions in the international version of the model: Belgium (BE), the rest of the Eurozone (EA) and the rest of the world (RW). Each region contains more or less the equations. The price variables are expressed in terms of the respective region's consumer price. However, since Belgium does not have an independent monetary authority, only EA and RW have Taylor rules. As a consequence, Belgian inflation is not a separate endogenous variable. Rather, a variable $pc_t^{BE} \equiv \frac{PC_t^{BE}}{PC_t^{EA}}$ is introduced that expresses the price of the BE numéraire in terms of the EA numéraire. Inflation in this variable can then be expressed as:

$$\frac{PC_t^{BE}}{PC_{t-1}^{BE}} = \frac{pc_t^{BE}}{pc_{t-1}^{BE}}\left(1 + \mathrm{inf}_{\mathrm{c,t}}^{EA}\right)$$

Note that pc_t^{BE} is a relative and stationary price.

7.1. The household block

The household maximizes an intertemporal utility function where the instantaneous utility function is of the form $(1-hab)\log(c_t - hab\ c_{t-1}) - \sum_{i=1}^{n}\sum_{k=l,m,h}\left(\frac{\omega_{ik}}{1+\frac{1}{\sigma_{ik}^{W}}} sl_i\ sk_{ik}\left(l_{ik,t}\right)^{1+\frac{1}{\sigma_{ik}^{W}}}\right)$, which is additively separable in consumption and labour. The labour supply part guarantees that under balanced growth, labour supply converges to an interior point rather than a corner solution. The household derives income from working, investing in both physical and R&D capital, which provides rental income, and holding government and foreign bonds. The first-order conditions (FOCs) are:

$\tilde{\lambda}_t^R = \dfrac{1-hab}{\tilde{c}_t^R - hab\ \tilde{c}_{t-1}^R}$	$R \in \{BE, EA, RW\}$
$\omega_{ik}^R\left(l_{ik,t}^R + \delta_{kh}\ lrd_{i,t}^R\right)^{\frac{1}{\sigma_{ik}^{W,R}}} = \tilde{\lambda}_t^R\left(\left(\dfrac{\theta_{ik}^{L,R}-1}{\theta_{ik}^{L,R}} - ben_{ik,t}^R\right)\left(1-\tau_{ik,t}^{L,R}\right)\tilde{w}_{ik,t}^R - \dfrac{\gamma_{ik}^{W,R}}{2}\dfrac{\theta_{ik}^{L,R}-1}{\theta_{ik}^{L,R}}\tilde{w}_{ik,t}^R\left(\dfrac{\tilde{w}_{ik,t}^R}{\tilde{w}_{ik,t-1}^R}-1\right)^2 + \dfrac{\gamma_{ik}^{W,R}}{\theta_{ik}^{L,R}}\tilde{w}_{ik,t}^R\left(\dfrac{\tilde{w}_{ik,t}^R}{\tilde{w}_{ik,t-1}^R}-1\right)\dfrac{\tilde{w}_{ik,t}^R}{\tilde{w}_{ik,t-1}^R}\right) - \beta\ \tilde{\lambda}_{t+1}^R\ (1 + npop)\left(\dfrac{\gamma_{ik}^{W,R}}{\theta_{ik}^{L,R}}\tilde{w}_{ik,t+1}^R\dfrac{l_{ik,t+1}^R + \delta_{kh}\ lrd_{i,t+1}^R}{l_{ik,t}^R + \delta_{kh}\ lrd_{i,t}^R}\left(\dfrac{\tilde{w}_{ik,t+1}^R}{\tilde{w}_{ik,t}^R}-1\right)\dfrac{\tilde{w}_{ik,t+1}^R}{\tilde{w}_{ik,t}^R}\right)$	$R \in \{BE, EA, RW\}$, $i \in \{1, \dots, n\}$, $k \in \{l, m, h\}$

$\frac{pc^{BE}_{t+1}}{pc^{BE}_t}\left(1+\mathrm{inf}^{EA}_{c,t+1}\right) = \frac{\beta\,\tilde\lambda^{BE}_{t+1}}{(1+\gamma_A)\,\tilde\lambda^{BE}_t}\left(1+inom^{BE}_t\right)$	
$1+\mathrm{inf}^{EA}_{c,t+1} = \frac{\beta\,\tilde\lambda^{EA}_{t+1}}{(1+\gamma_A)\,\tilde\lambda^{EA}_t}\left(1+inom^{EA}_t\right)$	
$1+inf^{RW}_{c,t+1} = \frac{\beta\,\tilde\lambda^{RW}_{t+1}}{(1+\gamma_A)\,\tilde\lambda^{RW}_t}\left(1+inom^{RW}_t\right)$	
$\left(1-taxcr^R_{i,t}\right) pi^R_{i,t} - q^R_{i,t}\left(1-\frac{\gamma^R_{i_i}}{2}\left(\frac{\tilde\iota^R_{i,t}}{\tilde\iota^R_{i,t-1}}-1\right)^2 - \gamma^R_{i_i}\left(\frac{\tilde\iota^R_{i,t}}{\tilde\iota^R_{i,t-1}}-1\right)\frac{\tilde\iota^R_{i,t}}{\tilde\iota^R_{i,t-1}}\right) = \beta\frac{\tilde\lambda^R_{t+1}}{\tilde\lambda^R_t}\,(1+npop)\,q^R_{i,t+1}\,\gamma^R_{i_i}\left(\frac{\tilde\iota^R_{i,t+1}}{\tilde\iota^R_{i,t}}-1\right)\left(\frac{\tilde\iota^R_{i,t+1}}{\tilde\iota^R_{i,t}}\right)^2$	$R\in\{BE,EA,RW\}$, $i\in\{1,\dots,n\}$
$q^R_{i,t} = \frac{\beta}{1+\gamma_A}\frac{\tilde\lambda^R_{t+1}}{\tilde\lambda^R_t}\left(pi^R_{i,t+1}\left(r^R_{i,t+1}\,ucap^R_{i,t+1}\left(1-\tau^{r,R}_{i,t+1}\right)-rprem^R_i - ucapa^R_i\left(ucap^R_{i,t+1}-ucapss^R_i\right) - ucapb^R_i\left(ucap^R_{i,t+1}-ucapss^R_i\right)^2\right)+\left(1-\delta^R_i\right)q^R_{i,t+1}\right)$	$R\in\{BE,EA,RW\}$, $i\in\{1,\dots,n\}$
$\left(1-rdtaxcr^R_{i,t}\right)\frac{prd^R_{i,t}}{1+npop} = \frac{\beta\tilde\lambda^R_{t+1}}{(1+\gamma_A)\,\tilde\lambda^R_t}\,prd^R_{i,t+1}\left(\left(1-\tau^{rd,R}_{i,t+1}\right)rrd^R_{i,t+1} + \left(1-\delta^R_A\right)\left(1-rdtaxcr^R_{i,t+1}\right)-rpremrd^R_i\right)$	$R\in\{BE,EA,RW\}$, $i\in\{1,\dots,n\}$
$r^R_{i,t}\left(1-\tau^R_{r,i,t}\right)-ucapa^R_i = 2\,ucapb^R_i\left(ucap^R_{i,t}-ucapss^R_i\right)$	$R\in\{BE,EA,RW\}$, $i\in\{1,\dots,n\}$
$\frac{pc^{BE}_{t+1}}{pc^{BE}_t}\left(1+\mathrm{inf}^{EA}_{c,t+1}\right) = \frac{\beta\,\tilde\lambda^{BE}_{t+1}}{(1+\gamma_A)\,\tilde\lambda^{BE}_t}\left(1+inom^{EA}_t - rprembf^{BE,EA}\left(\frac{\frac{1}{pc^{BE}_t}\widetilde{bf}^{BE,EA}_t}{\widetilde{gdp}^{BE}_t}-bftgt^{BE,EA}\right)\right)$	
$\frac{pc^{BE}_{t+1}}{pc^{BE}_t}\left(1+\mathrm{inf}^{RW}_{c,t+1}\right) = \frac{\beta\,\tilde\lambda^{BE}_{t+1}}{(1+\gamma_A)\,\tilde\lambda^{BE}_t}\frac{rer_{t+1}}{rer_t}\left(1+inom^{RW}_t - rprembf^{BE,RW}\left(\frac{\frac{rer_t}{pc^{BE}_t}\widetilde{bf}^{BE,RW}_t}{\widetilde{gdp}^{BE}_t}-bftgt^{BE,RW}\right)\right)$	

$$1+\inf_{c,t+1}^{RW} = \frac{\beta\,\tilde{\lambda}_{t+1}^{EA}}{(1+\gamma_A)\,\tilde{\lambda}_t^{EA}}\frac{rer_{t+1}}{rer_t}\left(1+inom_t^{RW} - rprembf^{EA,RW}\left(\frac{rer_t\,\widetilde{bf}_t^{EA,RW}}{\widetilde{gdp}_t^{EA}} - bftgt^{EA,RW}\right)\right)$$	

where the following accumulation relations have to be satisfied (note that the household income constraint is dropped invoking Walras' law):

$$\tilde{k}_{i,t}^R = (1-\delta_i^R)\frac{\tilde{k}_{i,t-1}^R}{(1+npop)(1+\gamma_A)} + \left(1-\frac{\gamma_{i_i^R}}{2}\left(\frac{\tilde{\imath}_{i,t}^R}{\tilde{\imath}_{i,t-1}^R}-1\right)^2\right)\tilde{\imath}_{i,t}^R$$	$(tca\ R\ i)$ $R\in\{BE,EA,RW\}$, $i\in\{1,\dots,n\}$
$$\tilde{A}_{i,t}^R = \frac{1-\delta_A}{1+\gamma_A}\,\tilde{A}_{i,t-1}^R + \widetilde{ird}_{i,t}^R$$	$(rdca\ R\ i)$ $R\in\{BE,EA,RW\}$, $i\in\{1,\dots,n\}$
$$\frac{1}{pc_t^{BE}}\widetilde{bf}_t^{BE,EA} = \left(1+inom_{t-1}^{EA} - rprembf^{BE,EA}\left(\frac{\frac{1}{pc_{t-1}^{BE}}\widetilde{bf}_{t-1}^{BE,EA}}{\widetilde{gdp}_{t-1}^{BE}} - bftgt^{BE,EA}\right)\right)\frac{\frac{1}{pc_t^{BE}}\widetilde{bf}_{t-1}^{BE,EA}}{(1+npop)(1+\gamma_A)\left(1+\inf_{c,t}^{EA}\right)} + \widetilde{ex}_t^{BE,EA} - \widetilde{im}_t^{BE,EA}$$	$(fca\ BE\ 1)$
$$\frac{rer_t}{pc_t^{BE}}\widetilde{bf}_t^{BE,RW} = \left(1+inom_{t-1}^{RW} - rprembf^{BE,RW}\left(\frac{\frac{rer_{t-1}}{pc_{t-1}^{BE}}\widetilde{bf}_{t-1}^{BE}}{\widetilde{gdp}_{t-1}^{BE}} - bftgt^{BE,RW}\right)\right)\frac{rer_t}{pc_t^{BE}}\frac{\widetilde{bf}_{t-1}^{BE,RW}}{(1+npop)(1+\gamma_A)\left(1+inf_{C,t}^{RW}\right)} + \widetilde{ex}_t^{BE,RW} - \widetilde{im}_t^{BE,RW}$$	$(fca\ BE\ 2)$
$$rer_t\,\widetilde{bf}_t^{EA,RW} = \left(1+inom_{t-1}^{RW} - rprembf^{EA,RW}\left(\frac{rer_{t-1}\,\widetilde{bf}_{t-1}^{EA,RW}}{\widetilde{gdp}_{t-1}^{EA}} - bftgt^{EA,RW}\right)\right)rer_t\,\frac{\widetilde{bf}_{t-1}^{EA,RW}}{(1+npop)(1+\gamma_A)\left(1+inf_{C,t}^{RW}\right)} + \widetilde{ex}_t^{EA,RW} - \widetilde{im}_t^{EA,RW}$$	$(fca\ EA)$

7.2. Production

The firm's production function, FOCs with respect to the 4 production factors and profit function are:

$$\tilde{y}_{i,t}^R = B_i^R\left(\tilde{k}_{G,t-1}^R\right)^{\alpha_{G,i}^R}\left(\frac{ucap_{i,t}^R\,\tilde{k}_{i,t-1}^R}{(1+npop)(1+\gamma_A)}\right)^{\alpha_i^R\left(1-\sigma_i^{M,R}\right)}$$	$R\in\{BE,EA,RW\}$, $i\in\{1,\dots,n\}$

$\left(\frac{\tilde{A}^R_{i,t-1}+\widetilde{GT}^R_{i,t-1}}{1+\gamma_A}\ sl^R_i\ l^R_{i,t}\right)^{(1-\alpha^R_i)\left(1-\sigma^{M,R}_i\right)} \tilde{m}^{\sigma^{M,R}_i}_{i,t}$	
$\frac{ucap^R_{i,t}\ \tilde{k}^R_{i,t-1}}{(1+npop)(1+\gamma_A)}=\frac{\alpha^R_i\left(1-\sigma^{M,R}_i\right)}{r^R_{i,t}\ pi^R_{i,t}}\ mc^R_{i,t}\ \tilde{y}^R_{i,t}$	$R\in\{BE,EA,RW\}$, $i\in\{1,\dots,n\}$
$\tilde{w}^R_{ik,t}=(1-\alpha^R_i)\left(1-\sigma^{M,R}_i\right) mc^R_{i,t}\frac{\tilde{y}^R_{ik,t}}{sl^R_i\ l^R_{i,t}}\ s^R_{l,ik}\left(\frac{sl^R_i\ sk^R_{ik}\ l^R_{ik,t}}{sl^R_i\ l^R_{i,t}}\right)^{-\frac{1}{\sigma^R_{li}}} - \gamma^{L,R}_{ik}\ \tilde{w}^R_{ik,t}\left(l^R_{ik,t}-l^R_{ik,t-1}\right)+\beta\frac{\tilde{\lambda}^R_{t+1}}{\tilde{\lambda}^R_t}\ \gamma^{L,R}_{ik}\ \tilde{w}^R_{ik,t+1}\left(l^R_{ik,t+1}-l^R_{ik,t}\right)$	$R\in\{BE,EA,RW\}$, $i\in\{1,\dots,n\}$, $k\in\{l,m,h\}$
$\tilde{m}^R_{i,t}=\frac{\sigma^{M,R}_i}{pm^R_{i,t}}\ mc^R_{i,t}\ \tilde{y}^R_{i,t}$	$R\in\{BE,EA,RW\}$, $i\in\{1,\dots,n\}$
$\frac{\tilde{A}^R_{i,t-1}+\widetilde{GT}^R_{i,t-1}}{1+\gamma_A}=\frac{(1-\alpha^R_i)\left(1-\sigma^{M,R}_i\right)}{rrd^R_{i,t}\ prd^R_{i,t}} mc^R_{i,t}\ \tilde{y}^R_{i,t}$	$R\in\{BE,EA,RW\}$, $i\in\{1,\dots,n\}$
$\widetilde{div}^R_{i,t}=p^R_{i,t}\ \tilde{y}^R_{i,t}\left(1-\left(\delta_{R,RW}+\delta_{R,EA}\right)\frac{\gamma^R_P}{2}\left(\frac{p^R_{i,t}}{p^R_{i,t-1}}\frac{1+inf^R_{c,t}}{1+\pi^*_c}-1\right)^2 - \delta_{R,BE}\frac{\gamma^R_P}{2}\left(\frac{p^{BE}_{i,t}}{p^{BE}_{i,t-1}}\frac{pc^{BE}_t}{pc^{BE}_{t-1}}\frac{1+inf^{EA}_{c,t}}{1+\pi^*_c}-1\right)^2\right) - r^R_{i,t}\ pi^R_{i,t}\frac{ucap^R_{i,t}\ \tilde{k}^R_{i,t-1}}{(1+npop)(1+\gamma_A)} - \sum_{k=l,m,h}\left(w^R_{ik,t}\ sl^R_i\ sk^R_{ik}\ l^R_{ik,t} + \frac{\gamma^{L,R}_{ik}}{2}\ w^R_{ik,t}\ sl^R_i\ sk^R_{ik}\left(l^R_{ik,t}-l^R_{ik,t-1}\right)^2\right) - pm^R_{i,t}\ \tilde{m}^R_{i,t} - rrd^R_{i,t}\ prd^R_{i,t}\frac{\tilde{A}^R_{i,t-1}}{1+\gamma_A}$	$R\in\{BE,EA,RW\}$, $i\in\{1,\dots,n\}$

Each industry's product is a CES aggregate of firm varieties. Because there are price adjustment costs, prices can deviate from their long-term value, which is marginal cost multiplied by a constant mark-up factor:

$(1-\sigma^{BE}_i)+\sigma^{BE}_i\frac{mc^{BE}_{i,t}}{p^{BE}_{i,t}}-\gamma^{BE}_P\left(\frac{p^{BE}_{i,t}}{p^{BE}_{i,t-1}}\frac{pc^{BE}_t}{pc^{BE}_{t-1}}\frac{1+inf^{EA}_{c,t}}{1+\pi^*_c}-1\right)\frac{p^{BE}_{i,t}}{p^{BE}_{i,t-1}}\frac{pc^{BE}_t}{pc^{BE}_{t-1}}\frac{1+inf^{EA}_{c,t}}{1+\pi^*_c} + \frac{\beta\ \tilde{\lambda}^{BE}_{t+1}}{\tilde{\lambda}^{BE}_t}\ \gamma^{BE}_P\left(\frac{p^{BE}_{i,t+1}}{p^{BE}_{i,t}}\frac{pc^{BE}_{t+1}}{pc^{BE}_t}\frac{1+inf^{EA}_{c,t+1}}{1+\pi^*_c} - 1\right)\left(\frac{p^{BE}_{i,t+1}}{p^{BE}_{i,t}}\frac{pc^{BE}_{t+1}}{pc^{BE}_t}\frac{1+inf^{EA}_{c,t+1}}{1+\pi^*_c}\right)^2(1+\pi^*_c)(1+npop)\frac{\tilde{y}^{BE}_{i,t+1}}{\tilde{y}^{BE}_{i,t}}=0$	$i\in\{1,\dots,n\}$
$(1-\sigma^R_i)+\sigma^R_i\frac{mc^R_{i,t}}{p^R_{i,t}}-\gamma^R_P\left(\frac{p^R_{i,t}}{p^R_{i,t-1}}\frac{1+inf^R_{c,t}}{1+\pi^*_c}-1\right)\frac{p^R_{i,t}}{p^R_{i,t-1}}\frac{1+inf^R_{c,t}}{1+\pi^*_c} + \frac{\beta\ \tilde{\lambda}^R_{t+1}}{\tilde{\lambda}^R_t}\ \gamma^R_P\left(\frac{p^R_{i,t+1}}{p^R_{i,t}}\frac{1+inf^R_{c,t+1}}{1+\pi^*_c}-1\right)\left(\frac{p^R_{i,t+1}}{p^R_{i,t}}\frac{1+inf^R_{c,t+1}}{1+\pi^*_c}\right)^2(1+\pi^*_c)(1+npop)\frac{\tilde{y}^R_{i,t+1}}{\tilde{y}^R_{i,t}}=0$	$R\in\{EA,RW\}$, $i\in\{1,\dots,n\}$

7.3. R&D

Industry-specific research institutes produce new knowledge employing researchers, subject to spillovers from other stocks of knowledge. After some time lag, newly produced knowledge becomes part of the “general technological level” of the economy. The production function, FOC, profit function and general technology accumulation function are of the form:

$\widetilde{ird}^R_{i,t} = \nu^R_i \prod_{K=BE,EA,RW}\prod_{j=1}^{n} \left(\frac{\tilde{A}^K_{j,t-1}}{1+\gamma_A}\right)^{\varphi^R_{ijK}} \left(sl^R_i\; sk^R_{ih}\; lrd^R_{i,t}\right)^{\lambda^R_{rd,i}}$	$R \in \{BE, EA, RW\}$, $i \in \{1, \dots, n\}$
$\lambda^R_{rd,i}\; prd^R_{i,t} \frac{\widetilde{ird}^R_{i,t}}{sl^R_i\; sk^R_{ih}\; lrd^R_{i,t}} - \gamma^{L,R}_{ird}\; \tilde{w}^R_{ih,t}\left(lrd^R_{i,t} - lrd^R_{i,t-1}\right) + \frac{\beta \tilde{\lambda}^R_{t+1}}{\tilde{\lambda}^R_t}\; \gamma^{L,R}_{ird}\; \tilde{w}^R_{ih,t+1}\left(lrd^R_{i,t+1} - lrd^R_{i,t}\right) = (1 - subs^R_i)\; \tilde{w}^R_{ih,t}$	$R \in \{BE, EA, RW\}$, $i \in \{1, \dots, n\}$
$\widetilde{divrd}^R_{i,t} = prd^R_{i,t}\; ird^R_{i,t} - (1 - subs^R_i)\; w^R_{ih,t}\; sl^R_i\; sk^R_{ih}\; lrd^R_{i,t} - \frac{\gamma^{L,R}_{ird}}{2} sl^R_i\; sk^R_{ih}\; w^R_{ih,t}\left(lrd^R_{i,t} - lrd^R_{i,t-1}\right)^2$	$R \in \{BE, EA, RW\}$, $i \in \{1, \dots, n\}$
$\widetilde{GT}^R_{i,t} = \tilde{A}^R_{i,t-tl}\; GT^R_{i,rel}$	$R \in \{BE, EA, RW\}$, $i \in \{1, \dots, n\}$

7.4. Demand aggregates

For each region in the model, there are $2n + 3$ demand aggregates (n being the number of industries): private consumption, public consumption and investment, and an investment and intermediate consumption good per industry. A nested demand aggregate X^R_t is a CES aggregate of products from different industries $X^R_{i,t}$. Under cost minimization, the optimal product quantities and aggregate price level are given by:

$X^R_{i,t} = s_{X^R,i}\left(\frac{P_{X^R_i,t}}{P_{X^R,t}}\right)^{-\sigma_{X^R}} X^R_t$	$R \in \{BE, EA, RW\}$, $i \in \{1, \dots, n\}$
$P_{X^R,t}^{1-\sigma_{X^R}} = \sum_{i=1}^{n}\left(s_{X^R,i}\left(P_{X^R_i,t}\right)^{1-\sigma_{X^R}}\right)$	$R \in \{BE, EA, RW\}$

These products are themselves CES aggregates of regional varieties, with corresponding FOCs and prices:

$X^{R,K}_{i,t} = s_{X^R_i,K}\left(\frac{\left(1+\tau^{R,K}_{X_i}\right)\left(1+\tau^{K,R}_{exp,X_i}\right) rer^{R,K}_t\; P^K_{i,t}}{P_{X^R_i,t}}\right)^{-\sigma_{X^R_i}} X^R_{i,t}$	$R, K \in \{BE, EA, RW\}$, $i \in \{1, \dots, n\}$
$P_{X^R_i,t}^{1-\sigma_{X^R_i}} = \sum_{K=BE,EA,RW}\left(s_{X^R_i,K}\left(\left(1+\tau^{R,K}_{X_i}\right)\left(1+\tau^{K,R}_{exp,X_i}\right) rer^{R,K}_t\; P^K_{i,t}\right)^{1-\sigma_{X^R_i}}\right)$	$R \in \{BE, EA, RW\}$, $i \in \{1, \dots, n\}$

7.5. Government

Government spends a fixed proportion of GDP on consumption and investment, its investment accumulating into public capital that increases the productivity in the private sector. It derives income from taxes on production factors and consumption, and pays for social benefits, R&D investment tax credits and wage subsidies for R&D workers. Government deficits accumulate into debt, which tends to a debt target in the long run. Structural changes are equilibrated by a debt-stabilizing lump sum tax. Central banks set the interest rate following a Taylor rule.

$pg_t^R\ \tilde{g}_t^R = ex_{g,t}^R\ \widetilde{gdp}_t^R$	$R \in \{BE, EA, RW\}$
$pig_t^R\ \tilde{ig}_t^R = ex_{ig,t}^R\ \widetilde{gdp}_t^R$	$R \in \{BE, EA, RW\}$
$\tilde{k}_{G,t}^R = \frac{1-\delta_G^R}{(1+npop)(1+\gamma_A)}\ \tilde{k}_{G,t-1}^R + \tilde{ig}_t^R$	$R \in \{BE, EA, RW\}$
$\tilde{b}_t^R = (1+inom_{t-1}^R)\frac{\tilde{b}_{t-1}^R}{(1+npop)\left(1+(1-\delta_{R,BE})inf_{c,t}^R+\delta_{R,BE}inf_{c,t}^{EA}\frac{pc_t^{BE}}{pc_{t-1}^{BE}}\right)(1+\gamma_A)} + pg_t^R\ \tilde{g}_t^R + pig_t^R\ \tilde{ig}_t^R + \sum_{i=1}^{n} taxcr_{i,t}^r\ pi_{i,t}^R\ \tilde{i}_{i,t}^R + \sum_{i=1}^{n} rdtaxcr_{i,t}^R\ prd_{i,t}^R\ ird_{i,t}^R + \sum_{i=1}^{n} subs_i^R\ \tilde{w}_{ih,t}^r\ sl_i^R\ sk_{ih}^r\ lrd_{i,t}^R + \sum_{i=1}^{n}\sum_{k=l,m,h} ben_{ik,t}^R\ \tilde{w}_{ik,t}^R\ (1-\tau_{ik,t}^{L,R})\ sl_i^r\ sk_{ik}^R\ (1-nonact_t^R-l_{ik,t}^R-\delta_{kh}\ lrd_{i,t}^R) - \sum_{i=1}^{n}\sum_{k=l,m,h} \tilde{w}_{ik,t}^R\ \tau_{ik,t}^{L,R}\ sl_i^R\ sk_{ik}^R\ (l_{ik,t}^R+\delta_{kh}\ lrd_{i,t}^R) - \sum_{i=1}^{n} r_{i,t}^R\ pi_{i,t}^R\ \tau_{i,t}^{r,R}\frac{ucap_{i,t}^R\ \tilde{k}_{i,t-1}^R}{(1+npop)(1+\gamma_A)} - \sum_{i=1}^{n}\tau_{i,t}^{rd,R} rrd_{i,t}^R\ prd_{i,t}^R\frac{\tilde{A}_{i,t-1}^R}{1+\gamma_A} - \sum_{i=1}^{n}\widetilde{div}_{i,t}^R\ \tau_{i,t}^{div,R} - \sum_{i=1}^{n}\widetilde{divrd}_{i,t}^R\ \tau_{i,t}^{divrd,R} - \sum_{j=1}^{n}\left(\sum_{K=BE,EA,RW} rer_t^{R,K} be_{pc,t}^{R,K}\ p_{j,t}^K\ (1+\tau_{j,R}^{exp,K})\left(\tau_{j,K}^{c,R}\ \tilde{c}_{j,K,t}^R+\tau_{j,K}^{g,R}\ \tilde{g}_{j,K,t}^R + \tau_{j,K}^{ig,R}\ \tilde{ig}_{j,K,t}^R+\sum_{i=1}^{n}\tau_{j,K}^{i_i,R}\ \tilde{i}_{ij,K,t}^R+\sum_{i=1}^{n}\tau_{j,K}^{m_i,R}\ \tilde{m}_{ij,K,t}^R\right) + \sum_{K\neq R}\left(p_{j,t}^R(1+\tau_{j,K}^{exp,R})\frac{GDP_0^K}{GDP_0^R}\left(\tilde{c}_{j,R,t}^K+\tilde{g}_{j,R,t}^K+\tilde{ig}_{j,R,t}^K+\sum_{i=1}^{n}\tilde{i}_{ij,R,t}^K+\sum_{i=1}^{n}\tilde{m}_{ij,R,t}^K\right)\right)\right) - \widetilde{tax}_t^R$	$R \in \{BE, EA, RW\}$
$\widetilde{tax}_t^R = \widetilde{tax}_{t-1}^R + tgov_1^R\left(\frac{\tilde{b}_t^R}{\widetilde{gdp}_t^R}-btgt^R\right)+tgov_2^R(\tilde{b}_t^R-\tilde{b}_{t-1}^R)$	$R \in \{BE, EA, RW\}$
$(1+inom_t^{EA}) = ilag^{EA}\ (1+inom_{t-1}^{EA}) + (1-ilag^{EA})\frac{1}{\beta}(1+\pi_c^*)(1+\gamma_A)\left(\frac{1+inf_{c,t}^{EA}}{1+\pi_c^*}\right)^{infsens}$	

$(1 + inom_t^{RW}) = ilag^{RW}\,(1 + inom_{t-1}^{RW}) + (1 - ilag^{RW})\frac{1}{\beta}(1+\pi_c^*)(1+\gamma_A)\left(\frac{1+inf_{c,t}^{RW}}{1+\pi_c^*}\right)^{infsens}$	

7.6. Resource constraints

This block contains the resource constraints for all products, taking adjustment costs into account. Also, import and export variables and GDP are defined.

$\left(1 - \frac{\gamma_P^R}{2}\left(\frac{p_{i,t}^R}{p_{i,t-1}^R}\frac{1+inf_{c,t}^R}{1+\pi_c^*}-1\right)^2 - \delta_{R,BE}\frac{\gamma_P^R}{2}\left(\frac{p_{i,t}^{BE}}{p_{i,t-1}^{BE}}\frac{pc_t^{BE}}{pc_{t-1}^{BE}}\frac{1+inf_{c,t}^{EA}}{1+\pi_c^*}-1\right)^2\right)\tilde{y}_{i,t}^R = \sum_{K=BE,EA,RW}\frac{GDP_0^K}{GDP_0^R}\left(\tilde{c}_{i,R,t}^K + \sum_{j=1}^n \tilde{\iota}_{ji,R,t}^K + \sum_{j=1}^n \widetilde{m}_{ji,R,t}^K + \tilde{g}_{i,R,t}^K + \widetilde{ig}_{i,R,t}^K\right) + \sum_{k=l,m,h}\frac{\gamma_{ik}^{W,R}}{2}\frac{\widetilde{w}_{ik,t}^R}{p_{i,t}}\, sl_i^R\, sk_{ik}^R\,(l_{ik,t}^R + \delta_{kh}\, lrd_{i,t}^R)\left(\frac{\widetilde{w}_{ik,t}^R}{\widetilde{w}_{ik,t-1}^R}\left(\delta_{R\neq BE} + \delta_{R=BE}\frac{pc_{t-1}^{BE}}{pc_t^{BE}}\right)-1\right)^2 + \sum_{k=l,m,h}\frac{\gamma_{ik}^{L,R}}{2}\frac{\widetilde{w}_{ik,t}^R}{p_{i,t}}\, sl_i^R\, sk_{ik}^R\,(l_{ik,t}^R - l_{ik,t-1}^R)^2 + \frac{\gamma_{ird}^{L,R}}{2}\frac{\widetilde{w}_{ih,t}^R}{p_{i,t}}\, sl_i^R\, sk_{ih}^R\,(lrd_{i,t}^R - lrd_{i,t-1}^R)^2 + \left(ucapa_i^R(ucap_{i,t}^R - ucapss_i^R) + ucapb_i^R(ucap_{i,t}^R - ucapss_i^R)^2\right)\frac{pi_{i,t}^R}{p_{i,t}^R}\frac{\tilde{k}_{i,t-1}^R}{(1+npop)(1+\gamma_A)}$	$R \in \{BE, EA, RW\}$, $i \in \{1, \dots, n\}$
$\widetilde{ex}_t^{R,K} = \sum_{j=1}^n\left(p_{j,t}^R\,(1+\tau_{j,K}^{exp,R})\frac{GDP_0^K}{GDP_0^R}\left(\tilde{c}_{j,R,t}^K + \tilde{g}_{j,R,t}^K + \widetilde{ig}_{j,R,t}^K + \sum_{i=1}^n \tilde{\iota}_{ij,R,t}^K + \sum_{i=1}^n \widetilde{m}_{ij,R,t}^K\right)\right)$	$R, K \in \{BE, EA, RW\}$
$\widetilde{im}_t^{R,K} = rer_t^{R,K}\, be_{pc,t}^{R,K}\sum_{j=1}^n p_{j,t}^K(1+\tau_{j,R}^{exp,K})\left(\tilde{c}_{j,K,t}^R + \tilde{g}_{j,K,t}^R + \widetilde{ig}_{j,K,t}^R + \sum_{i=1}^n \tilde{\iota}_{ij,K,t}^R + \sum_{i=1}^n \widetilde{m}_{ij,K,t}^R\right)$	$R, K \in \{BE, EA, RW\}$
$\widetilde{gdp}_t^R = \sum_{i=1}^n p_{i,t}^R\tilde{y}_{i,t}^R - \sum_{i=1}^n pm_{i,t}^R\widetilde{m}_{i,t}^R -$	$R \in \{BE, EA, RW\}$

8. List of variable and parameter definitions

8.1. Endogenous variables

Variables	Description	Variables	Description
$\tilde{A}^R_{i,t}$	Industry i's R&D capital stock in region R	$l^R_{ik,t}$	Employment rate of workers of skill level k in industry i in region R
$\widetilde{bf}^{R,K}_t$	Region R's holdings of region K's debt	$lrd^R_{i,t}$	R&D labour in industry i in region R
$\tilde{b}^R_t$	Real debt of region R	$\tilde{\lambda}^R_t$	Marginal utility of consumption in region R
B^R_i	Total factor productivity in industry i	$\tilde{m}^R_{it}$	Intermediate consumption goods used by industry i in region R
$\tilde{c}^R_t$	Aggregate household consumption in region R	$\tilde{m}^R_{ij,t}$	Intermediate consumption goods of product j used by industry i in region R
$\widetilde{div}^R_{i,t}$	Economic profits of industry i in region R	$\tilde{m}^R_{ij,K,t}$	Variety of intermediate goods j from region K used by industry i in region R
$\widetilde{divrd}^R_{i,t}$	Economic profits of the R&D sector in industry i in region R	$mc^R_{i,t}$	Real marginal costs of industry i in region R
$\widetilde{ex}^{R,K}_t$	Exports of region R to region K	$nonact^R_t$	Non active population in region R
$\widetilde{gdp}^R_t$	Real GDP per capita of region R	pc^{BE}_t	Ratio of consumer price deflator in BE with respect to EA
$\tilde{g}^R_t$	Aggregate government consumption in region R	$p^R_{g,t}$	Aggregate price of government consumption goods in region R
$\tilde{g}^R_{i,t}$	Government consumption of product i in region R	$p^R_{g_i,t}$	Aggregate price of government consumption of good i in region R
$\tilde{g}^R_{i,K,t}$	Public consumption of variety K of product i in region R	$pi^R_{i,t}$	Aggregate price of investment in industry i in region R
$\widetilde{GT}^R_{i,t}$	General technology of industry i in region R	$p^R_{i_{ij},t}$	Aggregate price of investment good j used by industry i in region R

Variables	Description	Variables	Description
$\tilde{i}_{i,t}^{R}$	Investment by industry i in region R	pig_t^R	Aggregate price of public investment in region R
$\tilde{i}_{ij,t}^{R}$	Investment of product j used by industry i in region R	$pig_{g_i,t}^R$	Aggregate price of government investment of good i in region R
$\tilde{i}_{ij,K,t}^{R}$	Variety of product j from region K invested by industry i in region R	$pm_{i,t}^R$	Aggregate price of intermediate consumption goods used by industry i in region R
$\widetilde{ig}_t^R$	Aggregate public investment of region R	$p_{m_{ij},t}^R$	Aggregate price of intermediate consumption goods j used by industry i in region R
$\widetilde{ig}_{i,t}^R$	Public investment of product i in region R	$prd_{i,t}^R$	Real price of the R&D good of industry i in region R
$\widetilde{ig}_{i,K,t}^R$	Public investment of variety K of product i of region R	$q_{i,t}^R$	Tobin's q of industry i in region R
$\widetilde{im}_t^{R,K}$	Imports of region R from region K	$r_{i,t}^R$	Capital rate of return of industry i in region R
$\mathrm{inf}_{c,t}^R$	Consumer inflation rate of region R	rer_t	Real exchange rate
$inom_t^R$	Nominal interest rate of region R	$rrd_{i,t}^R$	R&D capital rate of return of industry i in region R
$\widetilde{ird}_{i,t}^R$	Investment in R&D by industry i in region R	$ucap_{i,t}^R$	Capacity utilization rate of industry i in region R
$\tilde{k}_{i,t}^R$	Private capital stock in industry i in region R	$\tilde{w}_{ik,t}^R$	Gross real wage of workers of skill type k in industry i of region R
$\tilde{k}_{G,t}^R$	Aggregate public capital stock in region R	$\tilde{y}_{i,t}^R$	Aggregate output of industry i in region R
$l_{i,t}^R$	Labour input in industry i in region R		

8.2. Exogenous variables

Variables	Description
$ben_{ik,t}^{R}$	Unemployment benefit replacement rate in industry i in region R
$subs_{i}^{R}$	Implicit wage subsidy rate in industry i in region R
$\tau_{ik,t}^{L,R}$	The implicit tax rate on labour skill k in industry i in region R
$taxcr_{i,t}^{R}$	Tax credit on investment in industry i in region R
$\tau_{i,t}^{r,R}$	Implicit tax rate on capital income in industry i in region R
$rdtaxcr_{i,t}^{R}$	R&D tax credit in industry i in region R
$\tau_{i,t}^{rd,R}$	Implicit tax on R&D capital income in industry i in region R
$\tau_{i,t}^{div,R}$	Implicit tax rate on economic profits in industry i in region R
$\tau_{i,t}^{divrd,R}$	Implicit tax rate on economic profits of R&D in industry i in region R
$\tau_{j,K}^{c,R}$	Implicit product tax on household consumption good j from region K
$\tau_{j,K}^{g,R}$	Implicit product tax on government consumption good j from region K
$\tau_{j,K}^{m_i,R}$	Implicit product tax on intermediate consumption good j from region K used by industry i
$\tau_{j,K}^{i_i,R}$	Implicit product tax on investment good j from region K used by industry i
$\tau_{j,K}^{ig,R}$	Implicit tax on public investment goods j from region K
$\tau_{j,K}^{exp,R}$	Tax on exports of product j to region K
$\widetilde{tax}_{t}^{R}$	Real lump-sum tax in region R

8.3. Parameters

Parameters	Description	Parameters	Description
$\alpha_{G,i}^{R}$	Output elasticity of public capital	$s_{i_i,j}^{R}$	Share parameter of private investment by industry i in region R spent on product j
α_{i}^{R}	Combined with (1-σ_i^M) defines the output elasticity of capital	$s_{i_i,j,K}^{R}$	Share parameter of private investment by industry i in region R spent on regional variety K of product j
β	Discount factor	$s_{ig,i}^{R}$	Share parameter of public investment in region R spent on product i
$bftgt^{R,K}$	Debt target	$s_{ig,i,K}^{R}$	Share parameter of public investment in region R spent on regional variety K of product i
δ_{i}^{R}	Depreciation rate- tangible capital	$s_{m_i,j}^{R}$	Share parameter of intermediate good by industry i in region R spent on product j.

Parameters	Description	Parameters	Description
δ_A^R	Depreciation rate- intangible capital	$s_{m_{ij},K}^R$	Share parameter of intermediate good by industry i in region R spent on regional variety K of product j
γ_A	Technology growth rate	σ_c^R	Elasticity of substitution between consumption goods of different industries in region R
$\gamma_{ik}^{W,R}$	Wage adjustment costs parameter	σ_{ci}^R	Elasticity of substitution between regional varieties within consumption good i
$\gamma_{I_i}^R$	Investment adjustment costs parameter	σ_g^R	Elasticity of substitution between government consumption goods of different industries in region R
γ_P^R	Price adjustment costs parameter	σ_{gi}^R	Elasticity of substitution between regional varieties within government consumption good i
$\gamma_{ik}^{L,R}$	Labour adjustment costs parameter	$\sigma_{I_i}^R$	Elasticity of substitution between products within industry i's investment good in region R
$\gamma_{ird}^{L,R}$	Labour adjustment costs parameter – R&D sector	$\sigma_{I_{ij}}^R$	Elasticity of substitution between regional varieties of product j within industry i's investment good in region R
hab	Habit formation parameter	σ_{ig}^R	Elasticity of substitution between government investment goods of different industries in region R
$\lambda_{rd,i}^R$	Output elasticity of R&D labour	σ_{igi}^R	Elasticity of substitution between regional varieties within government investment good i
$ilag^R$	Persistence in the interest rate	$\sigma_{m_i}^R$	Elasticity of substitution between products within industry i's intermediate consumption good in region R
$infsens$	Response to inflation	$\sigma_{m_{ij}}^R$	Elasticity of substitution between regional varieties of product j within industry i's intermediate consumption good in region R
$npop$	Long-term population growth rate	$\sigma_i^{M,R}$	Output elasticity of intermediate consumption
φ_{ijK}^R	Spillover effects	$\sigma_{ik}^{W,R}$	Frisch labour supply elasticity of skill group k in industry i in region R
π_c^*	Long term inflation rate	$tgov_1^R$	Response to deficit in region R (debt correcting tax)

Parameters	Description	Parameters	Description
$rprem_i^R$	Capital risk premium of region R	$tgov_2^R$	Response to debt in region R (debt correcting tax)
$rpremrd_i^R$	Risk premium on R&D capital region R	$\theta_{ik}^{L,R}$	Elasticity of substitution between varieties of labour skill k in industry i in region R
$rprembf^{R,K}$	Risk premium on bonds from K region held by households of region R	$ucapa_i^R$	Capital utilization adjustment costs (linear) in in industry i region R
sl_i^R	Share of industry i in total labour force of region R	$ucapb_i^R$	Capital utilization adjustment costs (quadratic) in industry i in region R
$s_{l,ik}^R$	Efficiency of skill level k in industry i's labour process in region R	$ucapss_i^R$	Steady state capital utilization in industry i in region R
sk_{ik}^R	Share of skill *k* in the labour force of industry i	ν_i^R	Country specific efficiency constant in the production of new ideas in industry i
$s_{c,i}^R$	Share parameter of private consumption in region R spent on product i	ω_{ik}^R	Relative importance of labour supply with respect to consumption in industry i in region R
$s_{g,i}^R$	Share parameter of public consumption in region R spent on product i		